Cambridge Assessment International Education

Cambridge O Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

3463116585

ADDITIONAL MATHEMATICS **4037/22**

Paper 2 **May/June 2022**

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.

DC (PQ) 311245

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $\quad u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $\quad u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 DO NOT USE A CALCULATOR IN THIS QUESTION.

A curve has equation $y = \dfrac{6+\sqrt{x}}{3+\sqrt{x}}$ where $x \geqslant 0$. Find the exact value of y when $x = 6$. Give your answer in the form $a + b\sqrt{c}$, where a, b and c are integers. [3]

Substitute the value of $x = 6$

$$y = \frac{6+\sqrt{6}}{3+\sqrt{6}}$$

Introduce the conjugate as $3-\sqrt{6}$ and rationalise the surds.

$$y = \frac{6+\sqrt{6}}{3+\sqrt{6}} \times \frac{3-\sqrt{6}}{3-\sqrt{6}}$$

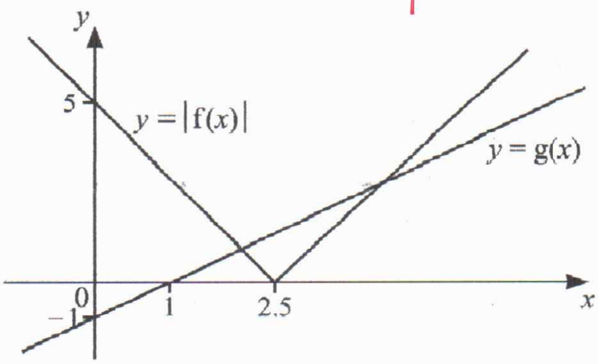
$$6(3-\sqrt{6}) + \sqrt{6}(3-\sqrt{6})$$

$$y = \frac{18 - 6\sqrt{6} + 3\sqrt{6} - 6}{9-6}$$

$$y = \frac{12 - 3\sqrt{6}}{3} \quad \text{(divide by 3)}$$

$$y = \underline{\underline{4 - \sqrt{6}}}$$

2



The diagram shows the graphs of $y = |\mathrm{f}(x)|$ and $y = \mathrm{g}(x)$, where $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$ are straight lines. Solve the inequality $|\mathrm{f}(x)| \leqslant \mathrm{g}(x)$. [5]

$f(x) = (0, 5), (2.5, 0)$

$$\text{Gradient} = \frac{\Delta y_2 - y_1}{x_2 - x_1} = \frac{0-5}{2.5-0} = \frac{-5}{2.5} = \underline{\underline{-2}}$$

equation of line $y = mx + c$ using points $(0, 5)$

$y = mx + c$

$5 = -2(0) + c$

$c = \underline{\underline{5}}$

equation of line $y = -2x + 5$

$f(x) = -2x + 5$

$g(x) = (1, 0), (0, -1)$

$$\text{Gradient} = \frac{-1-0}{0-1} = \underline{\underline{1}}$$

$y = mx + c$

$y = 1(0) + c$

$c = \underline{\underline{-1}}$

equation $y = x - 1$

$g(x) = x - 1$

$|f(x)| \leq g(x)$

$|-2x+5| \leq (x-1)$

$\pm(-2x+5) \leq (x-1)$

square both sides

$(-2x+5)^2 \leq (x-1)^2$

$4x^2 - 20x + 25 \leq x^2 - 2x + 1$

$4x^2 - x^2 - 20x + 2x + 25 - 1 \leq 0$

$3x^2 - 18x + 24 \leq 0$

divide by 3

$x^2 - 6x + 8 \leq 0$

$x^2 - 4x - 2x + 8 \leq 0$

$x(x-4) - 2(x-4) \leq 0$

$(x-4)(x-2) \leq 0$

$x = \underline{4}$ $x = \underline{2}$ $\underline{2 \leq x \leq 4}$

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3 Find the possible values of k for which the equation $kx^2 + (k+5)x - 4 = 0$ has real roots. [5]

For real roots discriminant ≥ 0

$b^2 - 4ac \geq 0$

$(k+5)^2 - 4(k)(-4) \geq 0$

$k^2 + 10k + 25 + 16k \geq 0$

$P = 25$
$S = 26$
$(25, 1)$

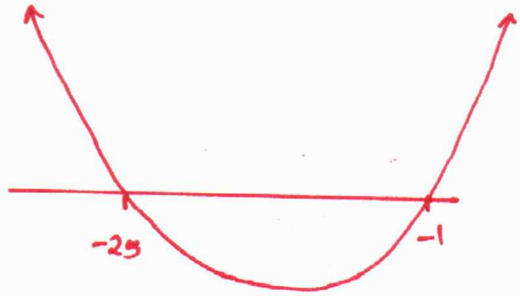
$k^2 + 26k + 25 \geq 0$

$k^2 + 25k + k + 25 \geq 0$

$k(k+25) + 1(k+25) \geq 0$

$(k+25)(k+1) \geq 0$

$k = -25 \quad k = -1$



$k \leq -25$ and $k \geq -1$

4 Variables x and y are related by the equation $y = 1 + \frac{2}{x} + \frac{1}{x^2}$ where $x > 0$. Use differentiation to find the approximate change in x when y increases from 4 by the small amount 0.01. [5]

At $x = 1$

$y = 1 + \frac{2}{x} + \frac{1}{x^2}$

$\frac{dy}{dx} = 0 + 2(-1)(x)^{-1-1} + (-2)x^{-2-1}$

$= \frac{-2}{x^2} - \frac{2}{x^3}$

$\left.\frac{dy}{dx}\right|_{x=1} = \frac{-2}{1} - \frac{2}{1}$

$= -4$

$\frac{0.01}{-4} = \frac{-1}{400}$

$5^3 = 125$

$5^4 = 625$

5 **(a)** Solve the equation $\dfrac{625^{\frac{x^3-1}{2}}}{125^{x^3}} = 5.$ [3]

Make the bases same, to the base 5

$$\frac{\left(5^4\right)^{\frac{x^3-1}{2}}}{\left(5^3\right)^{x^3}} = 5$$

$$\frac{5^{2x^3-2}}{5^{3x^3}} = 5$$

$$5^{2x^3-2} \div 5^{3x^3} = 5^1 \quad \text{(subtract powers)}$$

$$5^{-x^3-2} = 5^1$$

$$-x^3 - 2 = 1$$

$$-x^3 = 1 + 2$$

$$-x^3 = 3$$

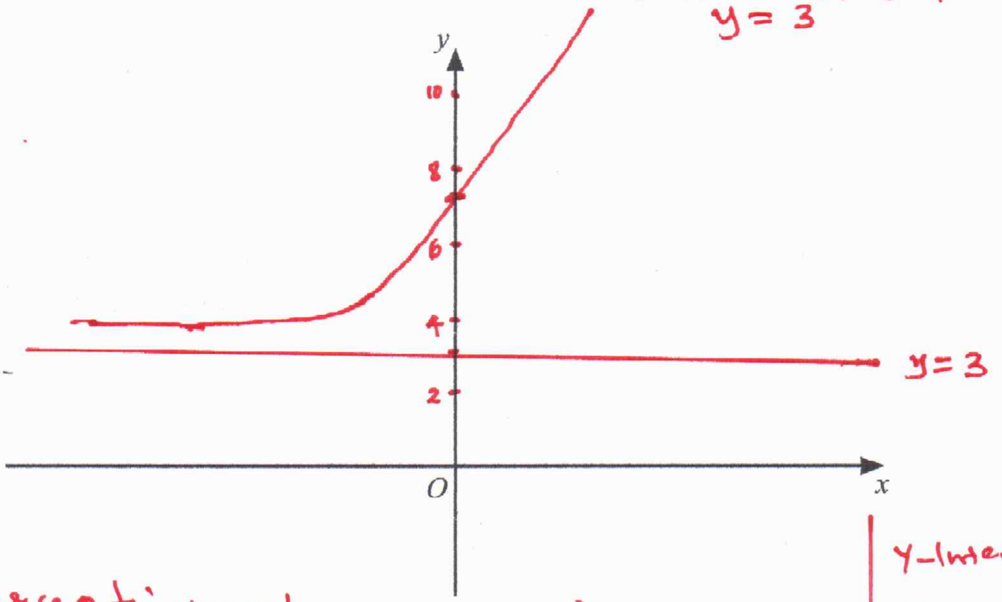
$$x = \sqrt[3]{-3}$$

(b) On the axes, sketch the graph of $y = 4e^x + 3$ showing the values of any intercepts with the coordinate axes. [2]

Horizontal asymptote $y = 3$

$x = 1$
$y = 13.87$

$x = -1$
$y = 4.$



For x-intercept; $y = 0$

$$y = 4e^x + 3$$

$$0 = 4e^x + 3$$

$$4e^x = -3$$

$$e^x = -3/4$$

$$\ln e^x = \ln(-3/4)$$

$$x = \ln(-3/4) \rightarrow \text{undefined}$$

(no-x-intercept)

Thus curve lies above.

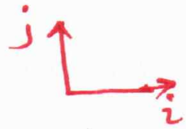
y-intercept $x = 0$

$$y = 4e^x + 3$$

$$y = 4e^0 + 3$$

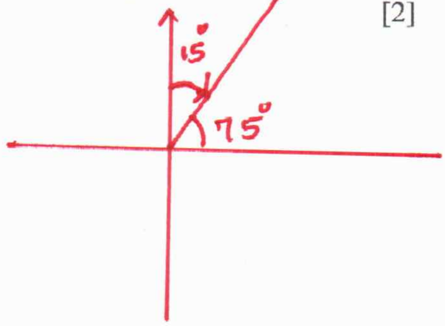
any number raised to zero is 1

$$y = 7$$



6 (a) In this question, **i** is a unit vector due east and **j** is a unit vector due north.

A cyclist rides at a speed of $4\,\text{ms}^{-1}$ on a bearing of 015°. Write the velocity vector of the cyclist in the form $x\mathbf{i} + y\mathbf{j}$, where x and y are constants. [2]



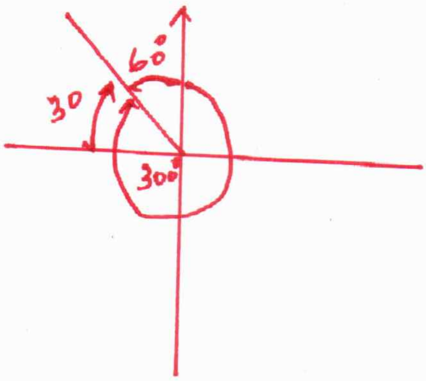
Since $x = r\cos\theta$ and $y = r\sin\theta$

$x = 4\cos 75°$, $y = 4\sin 75°$

$v = x\,i$ and $y\,j$ (unit vectors)

$v = \underline{4\cos 75°\, i}$ and $4\sin 75°\, j$

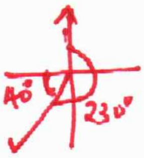
(b) A vector of magnitude 6 on a bearing of 300° is added to a vector of magnitude 2 on a bearing of 230° to give a vector **v**. Find the magnitude and bearing of **v**. [5]



$v_1 = x\underset{\sim}{i} + y\underset{\sim}{j}$

v_1 $x = -$ and $y = +$

$v_1 = -6\cos 30°\,\underset{\sim}{i} + 6\sin 30\,\underset{\sim}{j}$



$v_2 = x\underset{\sim}{i} + y\underset{\sim}{j}$

$v_2 = -2\cos 40°\underset{\sim}{i} - 2\sin 40°\underset{\sim}{j}$

Addition of v_1 and $v_2 = V$

$\left(-6\cos 30° - 2\cos 40°\right)i + \left(6\sin 30° - 2\sin 40°\right)j$

$\left(-6 \times \frac{\sqrt{3}}{2} - (-1.532)\right) \quad \left((6 \times \tfrac{1}{2}) - 1.28577\right) = 1.7144$

$= (-6.728)\underset{\sim}{i} + (1.7144)\underset{\sim}{j}$

Magnitude $|V| = \sqrt{(-6.728)^2 + (1.7144)^2}$

$|V| = \sqrt{45.265984 + 2.93916}$

$|V| = \sqrt{48.20515}$

$|V| = \underline{6.94 \text{ units}}$

bearing of $V = \tan\theta = \frac{x}{y}$

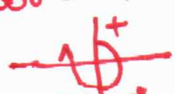
$\theta = \tan^{-1}\left(\frac{x}{y}\right)$

$\theta = \tan^{-1}\left(\frac{6.728}{1.7144}\right)$

$\theta = \underline{75.70}$

$180° - 75.70 = \underline{104.3°}$

$360° - 75.70 = 284.3°$



bearing of V

$= \underline{284.3}$

7 Differentiate $y = \dfrac{e^{4x}\tan x}{\ln x}$ with respect to x. [4]

Using Quotient rule.

$$\frac{d}{dx}\left(e^{4x}\tan x\right) = e^{4x}\sec^2 x + \tan x\left(4e^{4x}\right)$$
$$= e^{4x}\left(\sec^2 x + 4\tan x\right)$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}.$$

$$\frac{dy}{dx} = \frac{\ln x\, e^{4x}\left(\sec^2 x + 4\tan x\right) - e^{4x}\tan x\left(1/x\right)}{(\ln x)^2}$$
$$= \frac{x e^{4x}\ln x\left(\sec^2 x + 4\tan x\right) - e^{4x}\tan x}{x(\ln x)^2}$$

Since, $\sec^2 x = 1 + \tan^2 x$

$$= \frac{x e^{4x}\ln x\left(1 + \tan^2 x + 4\tan x\right) - e^{4x}\tan x}{x(\ln x)^2}$$

domain

8 The function f is defined by $f(x) = 3\sin^2 x - 2\cos x$ for $2 \leqslant x \leqslant 4$, where x is in radians.

(a) Find the x-coordinate of the stationary point on the curve $y = f(x)$. [5]

$f'(x) = 0$ (derivative)

$f(x) = 3\left(\frac{1-\cos 2x}{2}\right) - 2\cos x$

$f(x) = \frac{3}{2}(1 - \cos 2x) - 2\cos x$

$f'(x) \ \frac{3}{\cancel{2}}(\sin 2x \cdot \cancel{2}) - 2(-\sin x)$

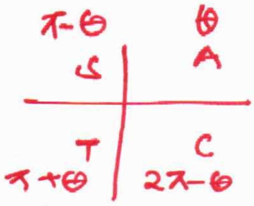
$= 3\sin 2x + 2\sin x$

$= 3(2\sin x \cos x) + 2\sin x$

$= 6\sin x \cos x + 2\sin x$

$= 2\sin x(3\cos x + 1)$

Since $2\sin x = 0$ and $3\cos x + 1 = 0$



$\sin x = 0$

$x = \sin^{-1}(0)$

$x = \underline{0}$

$x = \pi\ (3.142)$

Since it lies between the domain

$\cos x = \frac{-1}{3}$

$x = \cos^{-1}(-1/3)$

$x = \underline{1.231}$

$\pi - 1.231$

$= 1.91$ (does not belong to the domain)

$\pi + 1.231$

$= \underline{4.37}$ (above the domain)

The Point of $x = \underline{3.142}$.

(b) Solve the equation $f(x) = 1 - 3\cos x$. [5]

$3\sin^2 x - 2\cos x = 1 - 3\cos x$

$3(1-\cos^2 x) - 2\cos x - 1 + 3\cos x = 0$

$3 - 3\cos^2 x - 2\cos x - 1 + 3\cos x = 0$

$-3\cos^2 x + \cos x + 2 = 0$

$3\cos^2 x - \cos x - 2 = 0$ (quadratic form equation)

$3\cos^2 x - 3\cos x + 2\cos x - 2 = 0$

$3\cos x(\cos x - 1) + 2(\cos x - 1) = 0$

$(3\cos x + 2)(\cos x - 1) = 0$

$3\cos x + 2 = 0$

$\cos x = -2/3$

$x = \cos^{-1}(2/3)$

$x = \underline{0.841}$

$\pi - x = \underline{2.30}$

$\pi + x = \underline{3.98}$

$\cos x - 1 = 0$

$\cos x = 1$

$x = \cos^{-1}(1)$

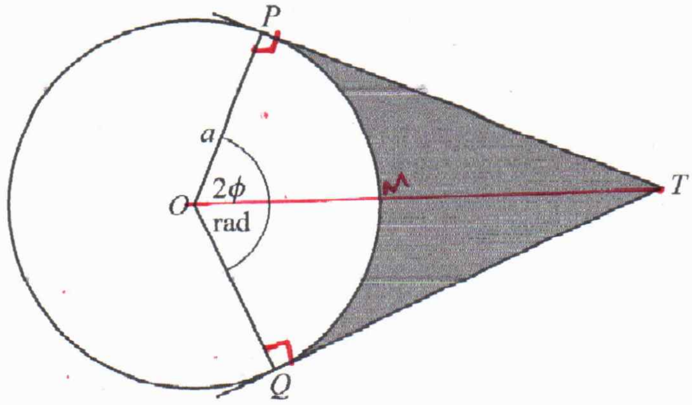
$x = \underline{0}$

$x = 2\pi$

$x = \underline{6.28}$ (Above the domain)

Values of $x = \underline{2.30}$ or $\underline{3.98}$

9 In this question all lengths are in centimetres.



The diagram shows a circle, centre O, radius a. The lines PT and QT are tangents to the circle at P and Q respectively. Angle POQ is 2ϕ radians.

(a) In the case when the area of the sector OPQ is equal to the area of the shaded region, show that $\tan\phi = 2\phi$. [4]

Area of sector $= \frac{1}{2}r^2\theta$

$= \frac{1}{\cancel{2}} a^2 (\cancel{2}\theta)$

$= \underline{a^2\theta}$

Area of triangle OPT

$\tan\phi = \frac{PT}{a}$

$PT = a\tan\phi$

Area of $\Delta OPT = \frac{1}{2}(a)(a\tan\phi)$

$= \underline{\frac{a^2}{2}(\tan\phi)}$

Area of sector POM $= \frac{a^2}{2}\theta$

Shaded region $= \frac{a^2}{2}\tan\phi - \frac{a^2}{2}\phi$

$= \underline{\frac{a^2}{2}(\tan\phi - \phi)}$

Area for whole shaded region

$= \frac{a^2}{2}(\tan\phi - \phi) \times 2$

$= \underline{a^2(\tan\phi - \phi)}$

$a^2\phi = a^2(\tan\phi - \phi)$

$a^2\phi = a^2\tan\phi - a^2\phi$

$a^2\phi + a^2\phi = a^2\tan\phi$

$2a^2\phi = a^2\tan\phi$

$\underline{\tan\phi = 2\phi}$

(b) In the case when the perimeter of the sector *OPQ* is equal to half the perimeter of the shaded region, find an expression for $\tan\phi$ in terms of ϕ. [3]

Perimeter of sector = $OP + OQ + PQ$

$= a + a + a2\phi$

$= 2a + 2a\phi$

Perimeter of shaded region = $PT + QT + PQ$

$= a\tan\phi + a\tan\phi + a2\phi$

$= 2a\tan\phi + 2a\phi$

$= 2a(\tan\phi + \phi)$

Perimeter of sector;

$2a + 2a\phi = \frac{\cancel{2}a}{\cancel{2}}(\tan\phi + \phi)$

$2\cancel{a}(1+\phi) = \cancel{a}(\tan\phi + \phi)$

$2 + 2\phi = \tan\phi + \phi$

$2 + 2\phi - \phi = \tan\phi$

$\tan\phi = 2 + \phi$

10 (a) A geometric progression has first term a and common ratio r, where $r > 0$. The second term of this progression is 8. The sum of the third and fourth terms is 160.

(i) Show that r satisfies the equation $r^2 + r - 20 = 0$. [4]

1st term $= a$,

2nd term $= 8$ (ar)

3rd term $= ar^2$

Since $\frac{a\cancel{r}}{\cancel{r}} = \frac{8}{r}$

$a = \frac{8}{r}$

Substitute $r = 8/r$

$ar^2 + ar^3 = 160$

$\frac{8}{r}(r^2) + \frac{8}{r}(r^3) = 160$

$8r + 8r^2 = 160$

Fourth term $= ar^3$

$8r^2 + 8r = 160$

$8r^2 + 8r - 160 = 0$

Divide by 8

$\frac{8r^2}{8} + \frac{8r}{8} - \frac{160}{8} = 0$

$\underline{\underline{r^2 + r - 20 = 0}}$

(ii) Find the value of a. [3]

$r^2 + r - 20 = 0$

Product $= -20$ $(5, -4)$

Sum $= +1$

$r^2 + 5r - 4r - 20 = 0$

$r(\cancel{r+5}) - 4(\cancel{r+5}) = 0$

$(r-4)(r+5) = 0$

$r - 4 = 0$ | $r + 5 = 0$

$r = \underline{4}$ | $r = \underline{\underline{-5}}$ (ignore negative value.

Substitute $r = 4$; $a = \frac{8}{r}$

$a = \frac{8}{4}$

$a = \underline{\underline{2}}$

$a = p$ $\quad d = 2$

(b) An arithmetic progression has first term p and common difference 2. The qth term of this progression is 14.
A different arithmetic progression has first term p and common difference 4. The sum of the first q terms of this progression is 168.

$a = p,\ d = 2$

Find the values of p and q. [6]

$a_n = a + (n-1)d$

$a_q = a + (q-1)d$

$p + (q-1)2 = 14$

$p + 2(q-1) = 14$

$p + 2q - 2 = 14$

$p + 2q = 16$ ... equation(i)

Sum of Progression $= S_n = \frac{n}{2}\{2a + (n-1)d]$

$168 = \frac{q}{2}(2p + (n-1)4]$

$q(p + 2(q-1)) = 168$ ... equation(ii)

Using equation(i) Make p the subject

$p + 2q = 16$

$p = 16 - 2q$

Using equation(ii) substitute p as $\underline{16-2q}$

$q(16 - \cancel{2q} + \cancel{2q} - 2) = 168$

$q(14) = 168$

$\frac{14q}{14} = \frac{168}{14}$

$q = \underline{\underline{12}}$

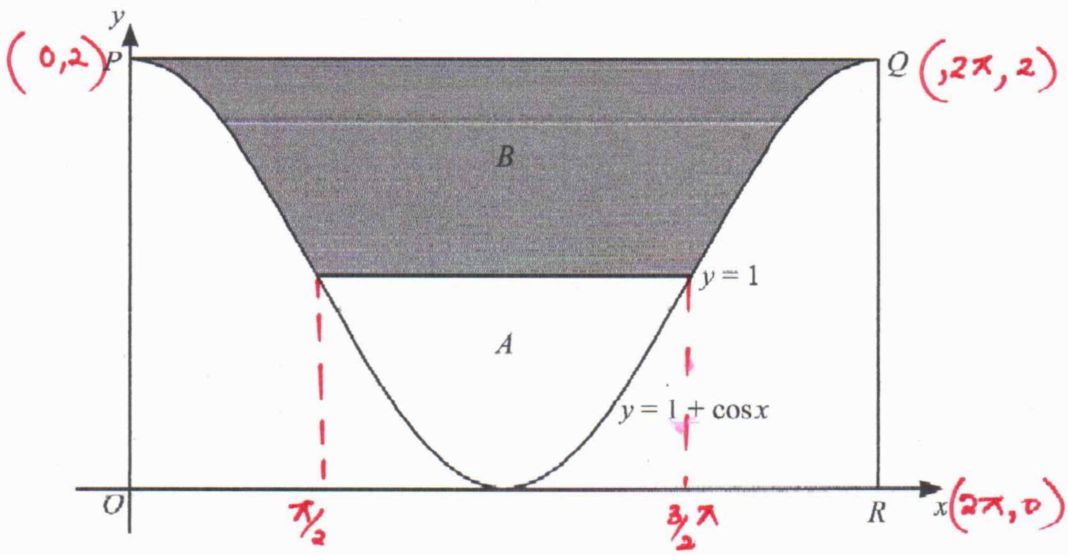
Substitue $q = 12$, $p = 16 - 2q$

$p = 16 - (2 \times 12)$

$p = 16 - 24$

$p = \underline{\underline{-8}}$

11



The diagram shows part of the line $y = 1$ and one complete period of the curve $y = 1 + \cos x$, where x is in radians. The line PQ is a tangent to the curve at P and at Q. The line QR is parallel to the y-axis. Area A is enclosed by the line $y = 1$ and the curve. Area B is enclosed by the line $y = 1$, the line PQ and the curve.

Given that area A : area B is $1 : k$ find the exact value of k. [9]

$y = 1 + \cos x$

Since 1 is amplitude

so $y = 2$; Co-ordinates $P(0,2)$

$y = 1 + \cos x$

$2 = 1 + \cos x$

$2 - 1 = \cos x$

$1 = \cos x$

$x = \cos^{-1}(1)$

$x = \underline{0}$

$R(2\pi, 0)$

Point of Intersection the line and curve.

$1 + \cos x = 1$

$\cos x = 1 - 1$

$\cos x = 0$

$x = \cos^{-1}(0) = \pi/2, \ 3/2\,\pi$

using Integration for line

$y = 1$

$\int_{\pi/2}^{3\pi/2} 1 \, dx = x \Big|_{\pi/2}^{3\pi/2} = \frac{3\pi}{2} - \frac{\pi}{2}$

Curve; $y = \int_{\pi/2}^{3\pi/2} (1 + \cos x)\, dx \ \cancel{=} \ (x + \sin x) \Big|_{\pi/2}^{3\pi/2}$

Continuation of working space for Question 11.

$$\left(x + \sin x\right)\Big|_{\frac{\pi}{2}}^{\frac{3}{2}\pi}$$

For the Curve; $\left(\frac{3\pi}{2} - \frac{\pi}{2}\right) + \left(\sin\frac{3\pi}{2} - \sin\frac{\pi}{2}\right)$

$= \left(\frac{3\pi}{2} - \frac{\pi}{2}\right) + (-1 - 1)$

$= -2 + \frac{3\pi}{2} - \pi/2.$

Line − curve; $\cancel{\frac{3}{2}}\pi - \cancel{\frac{\pi}{2}} + 2 - \cancel{\frac{3}{2}}\pi + \cancel{\frac{\pi}{2}}$

Area of A = $\underline{\underline{2}}$

Area of shape B;

Point of Intersection of line and Curve;

Since; $x = 0$, $x = 2\pi$

Area of rectangle = L×W

$= 2 \times 2\pi$

$= \underline{\underline{4\pi}}$

$$\int_0^{2\pi} (1 + \cos x)\,dx$$

$= \left(x + \sin x\right)\Big|_0^{2\pi}$

$= 2\pi + \sin 2\pi - 0 - \sin 0$

$= \underline{\underline{2\pi}}$

Area under PQ and the curve

$= 4\pi - 2\pi$

$= \underline{\underline{2\pi}}$

Area of B = $\underline{\underline{2\pi - 2}}$

Area A : Area B

2 : $2\pi - 2$

2 : $2(\pi - 1)$

1 : $\pi - 1$

value of k = $\underline{\underline{\pi - 1}}$

Question 12 is printed on the next page.

12 A curve is such that $\frac{d^2y}{dx^2} = \left(\frac{\sqrt{x}+1}{\sqrt[4]{x}}\right)^2$. Given that the gradient of the curve is $\frac{4}{3}$ at the point $(1, -1)$, find the equation of the curve. [7]

$(x)^{\frac{1}{4}\times 2} = x^{\frac{1}{2}}$

$$\frac{d^2y}{dx^2} = \frac{x + 2\sqrt{x} + 1}{\sqrt{x}}$$

$$= \frac{x}{\sqrt{x}} + \frac{2\sqrt{x}}{\sqrt{x}} + \frac{1}{\sqrt{x}}$$

$$= x^{\frac{1}{2}} + 2 + x^{-\frac{1}{2}}$$

$$\frac{d^2y}{dx^2} = x^{\frac{1}{2}} + 2 + x^{-\frac{1}{2}}$$

Integrate; $\int \frac{d^2y}{dx} = \int x^{\frac{1}{2}} + 2 + x^{-\frac{1}{2}}\,dx$

$$\frac{dy}{dx} = \frac{2}{3}x^{3/2} + 2x + 2x^{\frac{1}{2}} + C$$

Using gradient $4/3$ and $x = 1$

$$\frac{4}{3} = \frac{2}{3} + 2 + 2 + C$$

$$C = 4/3 - 2/3 - 4$$

$$C = \frac{-10}{3}$$

$$\frac{dy}{dx} = \frac{2}{3}x^{3/2} + 2x + 2x^{1/2} - \frac{10}{3}$$

$$\int \frac{dy}{dx} = \int \left(\frac{2}{3}x^{3/2} + 2x + 2x^{1/2} - 10/3\right)dx$$

$$y = \frac{2}{3}\cdot\frac{2x^{5/2}}{5} + 2\frac{x^2}{2} + 2\cdot\frac{2}{3}x^{3/2} - \frac{10}{3}x + D$$

$$y = \frac{4}{15}x^{5/2} + x^2 + 4/3\,x^{3/2} - \frac{10}{3}x + D$$

Substitute $y = -1$

$$-1 = 4/15 + 1 + 4/3 - 10/3 + D$$

$$D = -4/15$$

$$y = 4/15\,x^{5/2} + x^2 + \frac{4}{3}x^{3/2} - \frac{10}{3}x - 4/15$$