Cambridge Assessment International Education

Cambridge O Level

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

2664120921

ADDITIONAL MATHEMATICS **4037/11**

Paper 1 **May/June 2022**

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 Find constants a, b and c such that $\dfrac{\sqrt{p}\,q^{\frac{2}{3}}r^{-3}}{\left(pq^{-1}\right)^{2}r^{-1}} = p^{a}q^{b}r^{c}$. [3]

$$\frac{\sqrt{p}\;q^{2/3}\;r^{-3}}{(pq^{-1})^{2}\;r^{-1}}$$

$$\boxed{\sqrt{a} = a^{\frac{1}{2}}}$$

$$\frac{p^{1/2}\;q^{2/3}\;r^{-3}}{p^{2}\;q^{-2}\;r^{-1}}$$

$$p^{\frac{1}{2}} \div p^{2} \,.\, q^{2/3} \div q^{-2} ,\; r^{-3} \div r^{-1}$$

$$p^{1/2-2} \,.\, q^{2/3-(-2)} \,.\, r^{-3-(-1)}$$

$$p^{-3/2} \,.\, q^{8/3} \,.\, r^{-2}$$

$$= \underline{\underline{p^{-3/2}\;q^{8/3}\;r^{-2}}}$$

Constants a, b, c

$a = \underline{\underline{-3/2}}$

$b = 8/3$

$c = \underline{\underline{-2}}$

2 A particle moves in a straight line such that its displacement, s metres, from a fixed point, at time t seconds, $t \geqslant 0$, is given by $s = (1+3t)^{-\frac{1}{2}}$.

(a) Find the exact speed of the particle when $t = 1$. [3]

$$\frac{ds}{dt} = -\tfrac{1}{2}(1+3t)^{-\frac{1}{2}-1}\frac{d}{dt}(1+3t)$$

$$= -\tfrac{1}{2}(1+3t)^{-\frac{3}{2}}(3)$$

$$= -\tfrac{3}{2}(1+3t)^{-\frac{3}{2}}\Big|_{t=1}$$

speed $= \frac{3}{16}$

Substitute $t=1$

$$\left.\frac{ds}{dt}\right|_{t=1} = -\tfrac{3}{2}(1+3)^{-\frac{3}{2}}$$

$$= -\tfrac{3}{2}(4)^{-\frac{3}{2}}$$

$$= -\tfrac{3}{16}$$ (cannot be negative)

(b) Show that the acceleration of the particle will never be zero. [2]

Find the second derivative

$$\frac{d^2s}{dt^2} = -\tfrac{3}{2}\cdot -\tfrac{3}{2}(1+3t)^{-\frac{3}{2}-1}\frac{d}{dt}(1+3t)$$

$$= \tfrac{9}{4}(1+3t)^{-\frac{5}{2}}(3)$$

$$= \frac{27}{4}(1+3t)^{-\frac{5}{2}}$$ (acceleration)

Since a, cannot be zero;
$(1+3t)^{-5/2}$ is always positive
and acceleration never be zero.

3 A function f is such that $f(x) = \ln(2x+1)$, for $x > -\frac{1}{2}$.

(a) Write down the range of f. [1]

Range of $f(x) = \{f(x) \in \mathbb{R}\}$

A function g is such that $g(x) = 5x - 7$, for $x \in \mathbb{R}$.

(b) Find the exact solution of the equation $gf(x) = 13$. [3]

$gf(x) = g[\ln(2x+1)] = 5\ln(2x+1) - 7$

$5\ln(2x+1) - 7 = 13$

$5\ln(2x+1) = 13 + 7$

$\frac{\cancel{5}\ln(2x+1)}{\cancel{5}} = \frac{20}{5}$

$\ln(2x+1) = 4$

$e^{\ln(2x+1)} = e^4$

$2x+1 = e^4$

$2x = e^4 - 1$

$\frac{\cancel{2}x}{\cancel{2}} = \frac{e^4 - 1}{2}$

$x = \frac{1}{2}(e^4 - 1)$

(c) Find the solution of the equation $f'(x) = g^{-1}(x)$. [6]

$f'(x) = \frac{df}{dx} = \frac{d}{dx}(\ln(2x+1)] = \frac{1}{2x+1}\frac{d}{dx} = 2x+1 = \frac{2}{2x+1}$

Inverse;

$y = 5x - 7$

$x = 5y - 7$

$\frac{x+7}{5} = y$

$g^{-1}(x) = \frac{x+7}{5}$

$f'(x) = g^{-1}(x)$

$\frac{2}{2x+1} = \frac{x+7}{5}$

Cross multiply;

$2 \times 5 = (2x+1)(x+7)$

$10 = 2x^2 + 14x + x + 7$

$10 = 2x^2 + 15x + 7$

$2x^2 + 15x + 7 - 10 = 0$

$2x^2 + 15x - 3 = 0$

Using the quadratic equation formula;

$\frac{-15 \pm \sqrt{15^2 - 4 \times 2 \times (-3)}}{4}$

$\frac{-15 \pm \sqrt{225 + 24}}{4}$

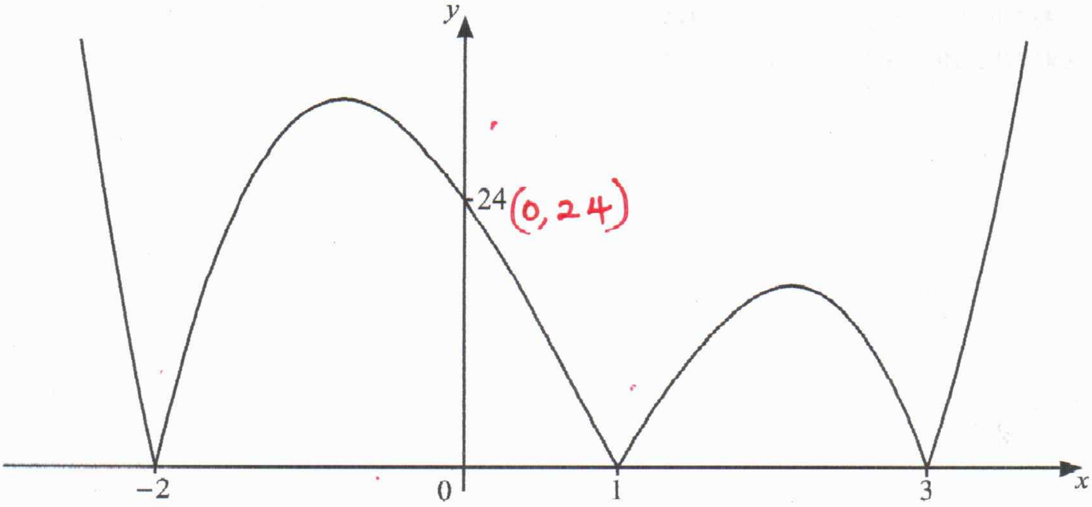
$-15 \pm$

$x = 0.195$

$x = -7.69$

Since $x > -\frac{1}{2}$; $x = 0.195$

4 (a)



The diagram shows the graph of $y = |f(x)|$, where f(x) is a cubic. Find the possible expressions for f(x). [3]

$f(x) = a(x-p)(x-q)(x-r)$

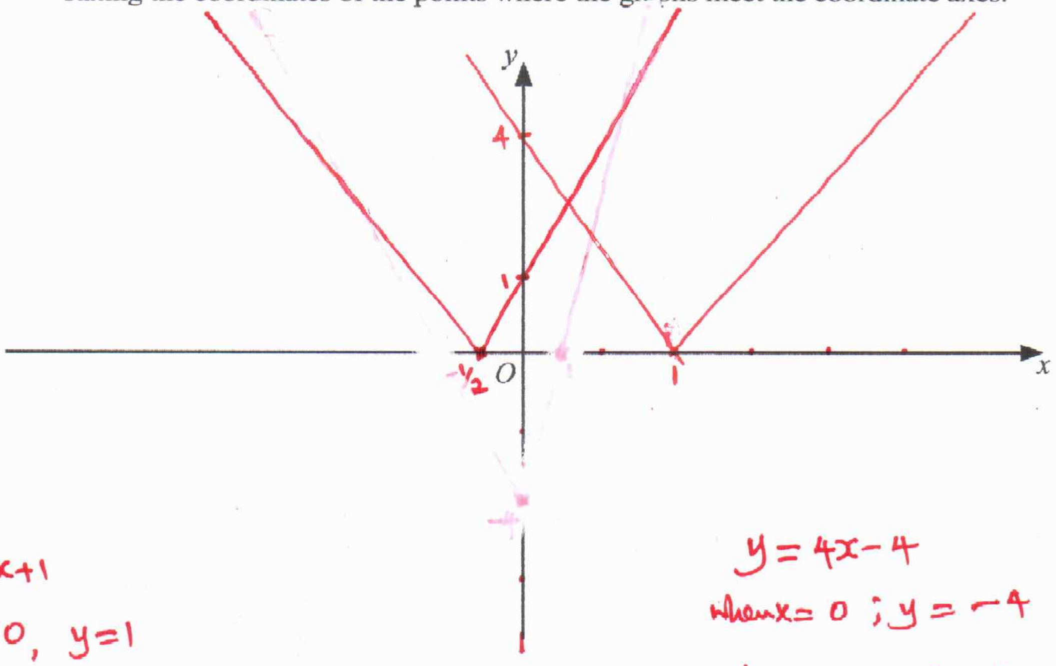
$f(x) = a(x+2)(x-1)(x-3)$

$24 = a(2)(-1)(-3)$

$\frac{24}{6} = \frac{6a}{6}$ $a = 4$

$f(x) = \pm 4(x+2)(x-1)(x-3)$

(b) (i) On the axes below, sketch the graph of $y = |2x+1|$ and the graph of $y = |4(x-1)|$, stating the coordinates of the points where the graphs meet the coordinate axes. [3]



$y = 2x+1$

When $x=0$, $y=1$

When $y=0$, $x = -\frac{1}{2}$ (-0.5)

$y = 4x-4$

When $x=0$; $y = -4$

When $x=1$, $y=0$

$y = |4(x-1)|$

When $y=0$

$\frac{0}{4} = \frac{4(x-1)}{4}$

$0 = x-1 \Rightarrow x = 1$

When $x=0$, $y = |4(0-1)| = |-4| = 4$

(ii) Find the exact solutions of the equation $|2x+1| = |4(x-1)|$. [4]

$|2x+1| = |4(x-1)|$

Square both sides.

$(2x+1)^2 = 16(x-1)^2$

$(2x+1)(2x+1)$

$2x(2x+1)+1(2x+1) = 16[(x-1)(x-1)]$

$4x^2+2x+2x+1 = 16[x^2-2x+1]$

$\underline{\underline{4x^2+4x+1}} = 16x^2-32x+16$

$4x^2+4x+1 = 16x^2-32x+16$

$4x^2-16x^2+4x+32x+1-16=0$

$-12x^2+36x-15=0$

$12x^2-36x+15=0$.

Divide by 3

$\frac{12x^2}{3} - \frac{36x}{3} + \frac{15}{3} = 0$

$4x^2-12x+5=0$.

Using quadratic equation formula;

$\frac{(-)-12 \pm \sqrt{(-12)^2-4(4)(5)}}{8}$

$\frac{12 \pm \sqrt{144-80}}{8}$

$\frac{12 \pm \sqrt{64}}{8}$

$x = \frac{12 \pm 8}{8}$

$x = \frac{12+8}{8}$ or $\frac{12-8}{8}$

$x = \frac{20}{8}$ $\quad$ $x\ 4/8 = \underline{\underline{0.5}}$

$x = \underline{\underline{2.5}}$

5 **(a)** Find the vector which is in the opposite direction to $\begin{pmatrix} 15 \\ -8 \end{pmatrix}$ and has a magnitude of 8.5. [2]

Magnitude

$|\vec{v}| = \sqrt{15^2 + (-8)^2}$

$= \sqrt{225 + 64}$

$= \sqrt{289}$

$= 17$

$17/2 = 8.5$

opposite direction

$v = -\frac{1}{2}\begin{pmatrix} 15 \\ -8 \end{pmatrix}$

(b) Find the values of a and b such that $5\begin{pmatrix} 3a \\ b \end{pmatrix} + \begin{pmatrix} 2a+1 \\ 2 \end{pmatrix} = 6\begin{pmatrix} b+a \\ 2 \end{pmatrix}$. [3]

$\begin{pmatrix} 15a \\ 5b \end{pmatrix} + \begin{pmatrix} 2a+1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6b+6a \\ 12 \end{pmatrix}$

$\begin{pmatrix} 15a + 2a+1 \\ 5b+2 \end{pmatrix} = \begin{pmatrix} 6b+6a \\ 12 \end{pmatrix}$

$\begin{pmatrix} 17a+1 \\ 5b+2 \end{pmatrix} = \begin{pmatrix} 6b+6a \\ 12 \end{pmatrix}$

$17a - 6a - 6b + 1 = 0$

$11a - 6b + 1 = 0$

$11a - 6b = -1$ ... equation (i)

$5b + 2 = 12$

$5b = 12 - 2$

$5b = 10$

$b = 2$

Substitute b as 2

$11a - 6b = -1$

$11a - 6(2) = -1$

$11a - 12 = -1$

$-a = -1$

$a = 1$

$a = 1$

$b = 2$

6 **(a)** Write down the values of k for which the line $y = k$ is a tangent to the curve $y = 4\sin\left(x + \frac{\pi}{4}\right) + 10$. [2]

amplitude + equation of curve;

Maximum $= a + c$

$= 4 + 10$

$= 14$

Minimum = equation of curve − amplitude

$= 10 - 4$

$= 6$

$y = 14$ or $y = 6$

(b) (i) Show that $\frac{1+\tan\theta}{1-\cos\theta}+\frac{1-\tan\theta}{1+\cos\theta}=\frac{2(1+\sin\theta)}{\sin^2\theta}$. [4]

$$=\frac{(1+\tan\theta)(1+\cos\theta)+(1-\tan\theta)(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$$

$$=\frac{1+\cancel{\cos\theta}+\cancel{\tan\theta}+\cos\theta\tan\theta+1-\cancel{\cos\theta}-\cancel{\tan\theta}+\cos\theta\tan\theta}{1-\cos^2\theta}$$

$$=\frac{2+2\cos\theta\tan\theta}{1-\cos^2\theta} \rightarrow \frac{2(1+\sin\theta)}{\sin^2\theta}$$

NB: $\boxed{\cos\theta\frac{\sin\theta}{\cos\theta}=\sin\theta \quad \cos^2\theta+\sin^2\theta=1 \quad \sin^2\theta=1-\cos^2\theta}$

(ii) Hence solve the equation $\frac{1+\tan\theta}{1-\cos\theta}+\frac{1-\tan\theta}{1+\cos\theta}=3$, for $0°\leqslant\theta\leqslant 360°$. [4]

$$\frac{2(1+\sin\theta)}{\sin^2\theta}=3$$

$$2(1+\sin\theta)=3\sin^2\theta$$

$$2+2\sin\theta=3\sin^2\theta$$

$$3\sin^2\theta-2\sin\theta-2=0$$

Using quadratic equation formula

$$\sin\theta=\frac{-(-2)\pm\sqrt{(-2)^2-4(3)(-2)}}{2(3)}$$

$$\sin\theta=\frac{2\pm\sqrt{4+24}}{6}$$

$$\sin\theta=\frac{2\pm\sqrt{28}}{6}$$

$$\sin\theta=1.2\cancel{1}525 \text{ (undefined)}$$

$$\frac{2-\sqrt{28}}{6}=-0.5485$$

$$\sin\theta=-0.5485$$

$$\sin^{-1}\theta=33.26°$$

3rd Quadrant $=180°+33.3 = 213.3°$

4th Quadrant $=360-33.3 = 326.7°$

7 **(a)** The first three terms of an arithmetic progression are $\lg 3$, $3\lg 3$, $5\lg 3$. Given that the sum to n terms of this progression can be written as $256\lg 81$, find the value of n. [5]

$S_n = \frac{n}{2}\{2a+(n-1)d)$

$d = a_2 - a_1$

$d = 3\lg 3 - \lg 3$

$= \underline{\underline{2\lg 3}}$

$\frac{n}{2}(2\lg 3 + (n-1)2\lg 3) = 256\lg 81.$

$\frac{n}{\cancel{2}}(\cancel{2\lg 3} + \cancel{2}n\lg 3 - \cancel{2\lg 3}) = 256\lg 81$

$n^2\lg 3 = 256\lg 81$

Since $3^4 = 81$

$n^2\lg 3 = 256\lg 3^4$

$n^2\cancel{\lg 3} = 256\times 4\cancel{\lg 3}$

$n^2 = 256\times 4$

$n^2 = 1024$

$n = \sqrt{1024}$

$n = \underline{\underline{32}}$

(b) DO NOT USE A CALCULATOR IN THIS PART OF THE QUESTION.

The first three terms of a geometric progression are $\ln 256$, $\ln 16$, $\ln 4$. Find the sum to infinity of this progression, giving your answer in the form $p \ln 2$. [4]

$S_\infty = \frac{a}{1-r}$

Common ratio $(r) = \frac{a_2}{a_1}$

$r = \frac{\ln 16}{\ln 256}$

$2^8 = 256$

$= \frac{1}{2}$

$S_\infty = \frac{\ln 256}{1 - 1/2}$

$S_\infty = \frac{\ln 2^8}{1/2}$

$S_\infty = 2 \times 8 \ln 2$

$S_\infty = 16 \ln 2$

$p = 16$

8 DO NOT USE A CALCULATOR IN THIS QUESTION.

(a) Find the exact coordinates of the points of intersection of the curve $y = x^2 + 2\sqrt{5}x - 20$ and the line $y = 3\sqrt{5}x + 10$. [4]

$y = x^2 + 2\sqrt{5x} - 20$ and $y = 3\sqrt{5x} + 10$

equate both curves.

$x^2 + 2\sqrt{5x} - 20 = 3\sqrt{5x} + 10$

$x^2 + 2\sqrt{5}x - 3\sqrt{5}x - 20 - 10 = 0$

$x^2 - \sqrt{5}x - 30 = 0$

Using Quadratic formula',

$x = \frac{-(-\sqrt{5}) \pm \sqrt{(-\sqrt{5})^2 - 4(1)(-30)}}{2}$

$x = \frac{\sqrt{5} \pm \sqrt{5 + 120}}{2}$

$x = \frac{\sqrt{5} \pm \sqrt{125}}{2}$

$\sqrt{125} = \sqrt{25 \times 5}$
$= 5\sqrt{5}$

$x = \frac{\sqrt{5} \pm 5\sqrt{5}}{2}$

$x = \frac{\sqrt{5}(1 + 5)}{2}$

$x = \frac{\sqrt{5}}{2}(1+5)$ or $\frac{\sqrt{5}}{2}(1-5)$

$= \frac{\sqrt{5}}{2}(6)$ $\qquad$ $\frac{\sqrt{5}}{2}(-4)$

$= 3\sqrt{5}$ $\qquad$ $= -2\sqrt{5}$

Co-ordinates

$(3\sqrt{5}, 55)$ and

$(-2\sqrt{5}, -20)$

$y = 3\sqrt{5x} + 10$

When $x = 3\sqrt{5}$

$y = 3\sqrt{5}(3\sqrt{5}) + 10$

$y = (3\sqrt{5})(3\sqrt{5}) + 10$

$y = (9 \times 5) + 10$

$y = 55$

When $y = 3\sqrt{5x} + 10$

When $x = -2\sqrt{5}$

$y = (3\sqrt{5})(-2\sqrt{5}) + 10$

$y = (-6 \times 5) + 10$

$y = -30 + 10$ $\quad$ $y = -20$

(b) It is given that $\tan\theta = \dfrac{\sqrt{3}-1}{2+\sqrt{3}}$, for $0 < \theta < \dfrac{\pi}{2}$. Find $\operatorname{cosec}^2\theta$ in the form $a + b\sqrt{3}$, where a and b are constants. [5]

$$\operatorname{Cosec}^2\theta = 1 + \cot^2\theta$$

$$\cot\theta = \frac{2+\sqrt{3}}{\sqrt{3}-1}$$

$$\operatorname{Cosec}^2\theta = 1 + \left(\frac{2+\sqrt{3}}{\sqrt{3}-1}\right)^2$$

$$= \frac{(\sqrt{3}-1)^2 + (2+\sqrt{3})^2}{(\sqrt{3}-1)^2}$$

$$= \frac{3-2\sqrt{3}+1+4+4\sqrt{3}+3}{3-2\sqrt{3}+1}$$

$$= \frac{2\sqrt{3}+11}{4-2\sqrt{3}}$$

Rationalize the denominator by Conjugate;

$$\frac{2\sqrt{3}+11}{4-2\sqrt{3}} \times \frac{4+2\sqrt{3}}{4+2\sqrt{3}}$$

$$= \frac{(2\sqrt{3}+11)(2\sqrt{3}+4)}{(4^2)-(2\sqrt{3})^2}$$

$$= \frac{4\times3+8\sqrt{3}+22\sqrt{3}+44}{16-4\times3}$$

$$= \frac{12+8\sqrt{3}+22\sqrt{3}+44}{4}$$

$$\frac{30\sqrt{3}+56}{4}$$

$$= \frac{56}{4} + \frac{30\sqrt{3}}{4}$$

$$= 14 + \frac{15}{2}\sqrt{3}$$

9 A circle, centre O and radius r cm, has a sector OAB of fixed area $10\,\text{cm}^2$. Angle AOB is θ radians and the perimeter of the sector is P cm.

(a) Find an expression for P in terms of r. [3]

Area of sector $= \frac{1}{2}r^2\theta$

$\frac{1}{2}r^2\theta = 10$

$\theta = \frac{20}{r^2}$

Perimeter $= OA + OB + AB$

$= r + r + r\theta$

$2r + r\left(\frac{20}{r^2}\right)$

$2r + \frac{20}{r}$

$P = 2r + \frac{20}{r}$

(b) Find the value of r for which P has a stationary value. [3]

$\frac{dP}{dr} = 2 - \frac{20}{r^2} = 0$

$\frac{20}{r^2} = \frac{2}{1}$

$\frac{20}{2} = \frac{2r^2}{2}$

$10 = r^2$

$r^2 = 10$

$r = \sqrt{10}$

(c) Determine the nature of this stationary value. [2]

$\frac{d^2P}{dr^2} = \frac{d}{dr}\left(2 - \frac{20}{r^2}\right)$

$= -20\left(-\frac{2}{r^3}\right)$ Positive values means

$= \frac{40}{r^3} > 0$ (minimum)

(d) Find the value of θ at this stationary value. [1]

stationary value $= \sqrt{10}$, $\theta = \frac{20}{r^2}$

$= \frac{20}{(\sqrt{10})^2} = \frac{20}{10}$ $\theta = 2$

10 The normal to the curve $y = \tan\left(3x + \frac{\pi}{2}\right)$ at the point P with coordinates $(p, -1)$, where $0 < p \leqslant \frac{\pi}{6}$, meets the x-axis at the point A and the y-axis at the point B. Find the exact coordinates of the mid-point of AB. [10]

Co-ordinates of Point P $(p, -1)$

$y = \tan\left(3x + \frac{\pi}{2}\right)$

$-1 = \tan\left(3p + \pi/2\right)$

$3p + \pi/2 = \tan^{-1}(1)$

$3p + \pi/2 = \pi/4$

$\theta = \pi/4$

2nd Quadrant $= 3p + \pi/2 = \pi - \pi/4$

$3p + \pi/2 = 3/4\,\pi$

$3p = 3/4\,\pi - \pi/2$

$3p = \pi/4$

$12p = \pi$

$p = \pi/12$ (Belongs to the domain)

4th Quadrant $= 3p + \pi/2 = 7/4\,\pi$

$3p = 7/4\,\pi - \pi/2$

$3p = 5\pi/4$

$p = \frac{5}{12}\pi$ (Does not belong to domain).

Co-ordinate of $P = (\pi/12, -1)$

Find equation of normal

$= y - y_0 = \frac{-1}{m}(x - x_0)$

$(x_0, y_0) = (\pi/12, -1)$

$\pi-\theta$	θ
$\pi+\theta$	$2\pi-\theta$

$m = \left.\frac{dy}{dx}\right|_p = \sec^2\left(3x + \pi/2\right) \times 3$

$m|_p = 3\sec^2\left(3\frac{\pi}{12} + \pi/2\right)$

$= 3\sec^2 3\pi/4$

$= \frac{3}{\cos 3\pi/4} = \frac{3}{\frac{1}{2}} = 3 \times \frac{2}{1}$

$= 6$

$y - (-1) = -1/6\left(x - \pi/12\right)$

$y + 1 = -1/6\left(x - \pi/12\right)$

At x-axis $y = 0$

$1 = -1/6\left(x - \pi/12\right)$

$x - \pi/12 = -6$

$x = -6 + \pi/12$ $\quad A\left(\pi/12 - 6, 0\right)$

At y-axis $x = 0$

$y + 1 = -1/6\left(x - \pi/12\right)$

$y + 1 = -1/6\left(-\pi/12\right)$

$y = \pi/72 - 1$ $\quad B\left(0, \pi/72 - 1\right)$

Points Co-ordinates $A\left(\pi/12 - 6, 0\right)$

$B\left(0, \pi/72 - 1\right)$

Midpoint of AB = $\left(\dfrac{\frac{\pi}{12}-6}{2}, \dfrac{\frac{\pi}{72}-1}{2}\right)$

$= \left(\dfrac{\pi-72}{24}, \dfrac{\pi-72}{144}\right)$

Simplifying Further; $\left(\frac{\pi}{24} - \frac{72}{24}, \frac{\pi}{144} - \frac{72}{144}\right)$

$= \left(\dfrac{\pi}{24} - 3, \dfrac{\pi}{144} - \dfrac{1}{2}\right)$