Algebra and graphs – 2023 O Level Math D 4024
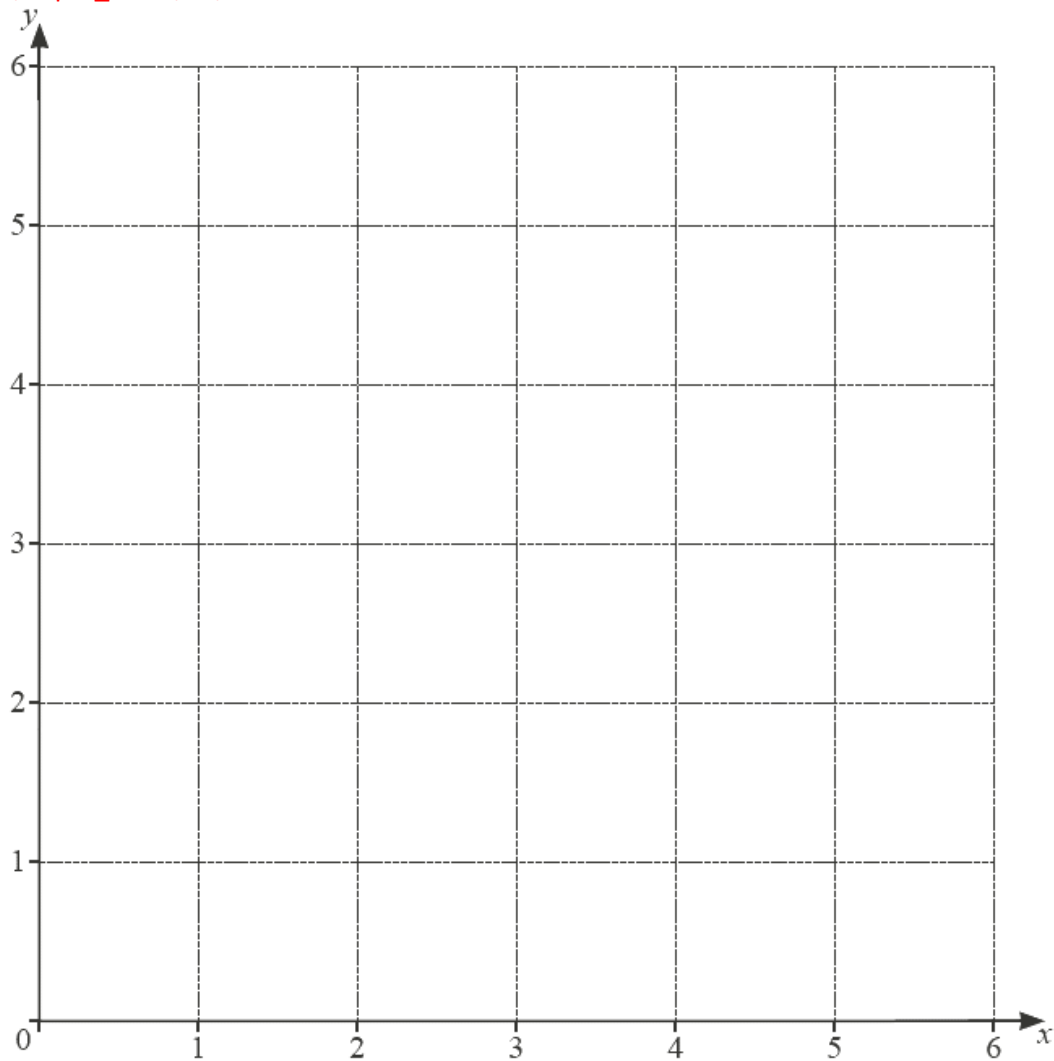
1. Nov/2023/Paper_ 4024/11/No.16

The region R is defined by these inequalities.

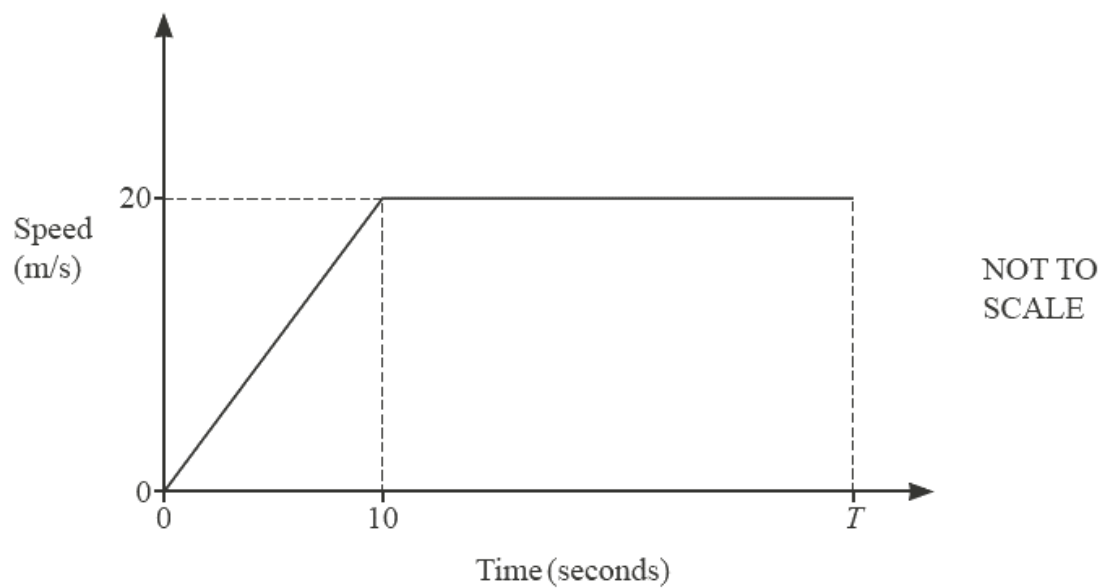$$1 \leqslant x \leqslant 3$$
$$2 \leqslant y \leqslant 3$$
$$y \geqslant \frac{x}{2} + 1$$

Find and label region R. [4]

1

2. Nov/2023/Paper_ 4024/11/No.19

The diagram shows the speed–time graph for part of a car journey.

(a) Calculate the acceleration of the car in the first 10 seconds of the journey.

.... m/s^2 [1]

(b) The car travels $700\,\text{m}$ in T seconds.

Find the value of T.

$T =$ .... [3]

3. Nov/2023/Paper_ 4024/11/No.21

(a) Factorise $6a - 9$.

.... [1]

(b) Factorise $4b^2 - 25$.

.... [1]

(c) Simplify $\dfrac{2c^2 - 8c}{2c^2 - 5c - 12}$.

.... [3]

3. Nov/2023/Paper_ 4024/11/No.21

4. Nov/2023/Paper_ 4024/11/No.22

$$f(x) = \frac{x}{4} + 3 \qquad\qquad g(x) = 2(x-1)$$

(a) Find $f(-8)$.

.... [1]

(b) Find $f^{-1}(x)$.

$f^{-1}(x) =$ .... [2]

(c) $f(p) = g(p+5)$

Find the value of p.

$p =$ .... [3]

5. Nov/2023/Paper_ 4024/11/No.24

Solve $\dfrac{3x}{x+1} - \dfrac{2}{x-1} = 3$.

$x =$ .... [4]

5. Nov/2023/Paper_ 4024/11/No.24

Solve $\dfrac{3x}{x+1} - \dfrac{2}{x-1} = 3$.

6. Nov/2023/Paper_ 4024/12/No.11

Solve $7m - 13 \leqslant 8$.

.... [2]

7. Nov/2023/Paper_ 4024/12/No.12

Solve the simultaneous equations.
Show all your working.

$$5x + 4y = 14$$
$$3x - 2y = 15$$

$x = $ ....

$y = $ .... [3]

6. Nov/2023/Paper_ 4024/12/No.11

8. Nov/2023/Paper_ 4024/12/No.15

(a) The second term of a linear sequence is 28.
The fifth term of the sequence is 16.

Find the first term, the third term and the fourth term of this sequence.

First term = ...

Third term = ...

Fourth term = ... [2]

(b) These are the first five terms of a different sequence.

$$3 \qquad 9 \qquad 19 \qquad 33 \qquad 51$$

Find an expression for the nth term of this sequence.

.... [2]

9. Nov/2023/Paper_ 4024/12/No.16

$$T = \sqrt{P-4}$$

(a) Work out the value of T when $P = 40$.

$T = $ .... [1]

(b) Rearrange the formula to make P the subject.

$P = $ .... [2]

9. Nov/2023/Paper_ 4024/12/No.16

10. Nov/2023/Paper_ 4024/12/No.18

The diagram shows the speed–time graph of part of a journey for two cyclists, A and B.
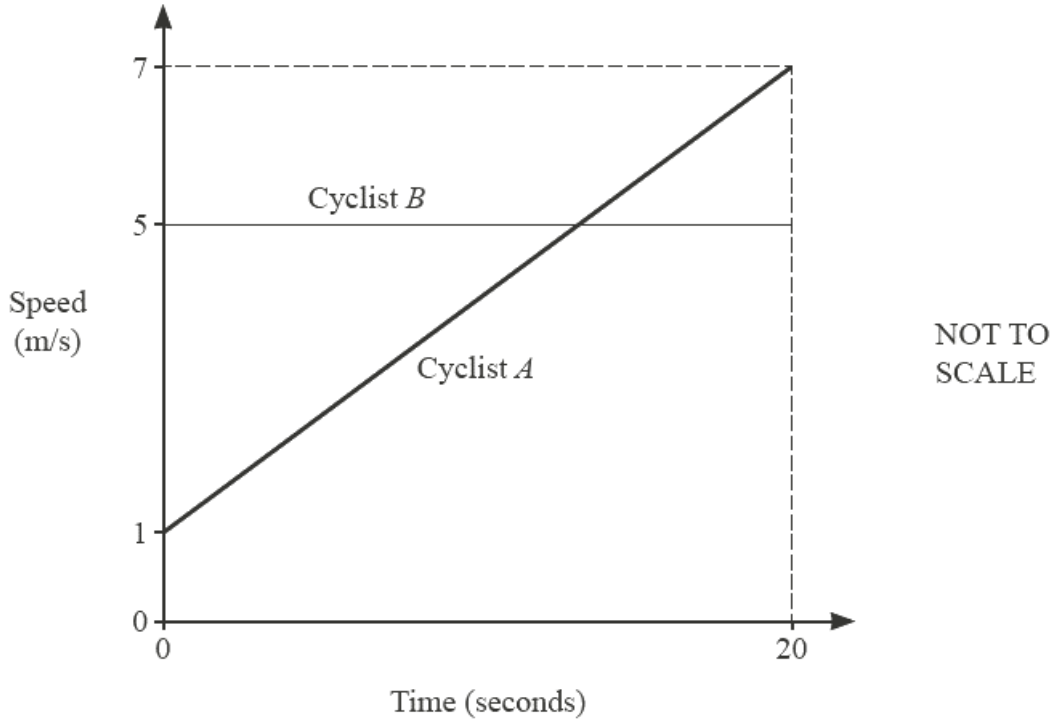
(a) Find the acceleration of cyclist A during the first 20 seconds.

.... m/s^2 [1]

(b) Find which cyclist travelled further in the first 20 seconds and by how many metres.

Cyclist .... travelled further by .... metres [3]

11. Nov/2023/Paper_ 4024/12/No.19

Express as a single fraction in its simplest form.

$$\frac{x+1}{8} + \frac{3x}{4} - \frac{5x}{16}$$

.... [2]

12. Nov/2023/Paper_ 4024/12/No.20

Factorise.

(a) $2cd + ce - 6d - 3e$

.... [2]

(b) $3v^2 - 27t^2$

.... [2]

13. Nov/2023/Paper_ 4024/21/No.4

(a) These are the first five terms of a sequence.

$$5 \quad 12 \quad 19 \quad 26 \quad 33$$

(i) Find the next term of the sequence.

.... [1]

(ii) Find an expression for the nth term of the sequence.

.... [2]

(iii) T_k and T_{k+1} are consecutive terms of the sequence.
The sum of these two terms is 703.

Find the value of k.

$k =$ .... [3]
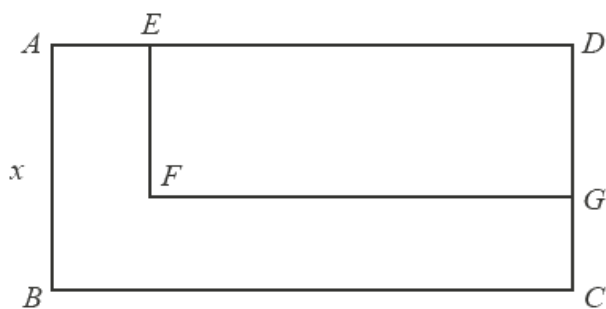
(b) An expression for the nth term of a different sequence is $n^2 + an + b$.
The 1st term of the sequence is 3.
The 3rd term of the sequence is 19.

Find the 6th term of the sequence.

.... [5]

14

14. Nov/2023/Paper_ 4024/21/No.6

$ABCD$ is a rectangle with area $30\,\text{cm}^2$.
$AB = x$ cm.

Rectangle $DEFG$ is removed from the corner of rectangle $ABCD$.
$AE = CG = 2$ cm.

(a) Write down an expression for BC in terms of x.

.... [1]

(b) Show that the shaded area, $y\,\text{cm}^2$, is given by
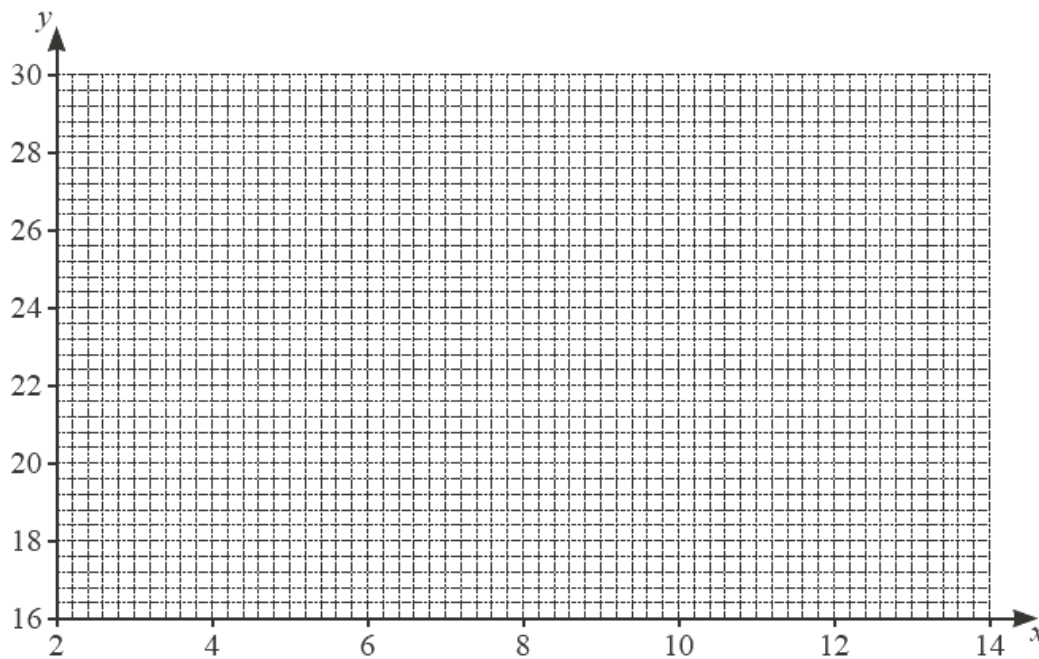
$$y = 2x + \frac{60}{x} - 4.$$

[3]

15

(c) Complete the table for $y = 2x + \dfrac{60}{x} - 4$.

Write your answer correct to 1 decimal place.

x	2	3	4	6	8	10	12	14
y	30	22	19	18	19.5	22	25	

[1]

(d) Draw the graph of $y = 2x + \dfrac{60}{x} - 4$ for $2 \leqslant x \leqslant 14$.

[3]

(e) The shaded area is $24 \, \text{cm}^2$.
The length of AB is less than the length of BC.

Use your graph to find the dimensions of rectangle $ABCD$.

.... cm by .... cm [2]

15. Nov/2023/Paper_ 4024/21/No.7

(a) Simplify.

$$7a - 4b - 2a + b$$

.... [2]

(b) Expand and simplify.

$$3(2x - 3) + 5(x + 2)$$

.... [2]

(c) Solve.

$$6x^2 - 2x - 9 = 0$$

Show all your working and give your answers correct to 2 decimal places.

$x =$ .... or $x =$ .... [3]

(d) Write as a single fraction in its simplest form.

(i) $\dfrac{x}{4} \div \dfrac{2}{y}$

.... [1]

(ii) $\dfrac{3}{x-5} - \dfrac{7}{2x+1}$

.... [3]

16. Nov/2023/Paper_ 4024/22/No.4

(a) Solve.

$$5x + 6 = 3x$$

$x =$ .... [2]

(b) The mass of a plum is n grams.
The mass of an apple is twice the mass of the plum.
The mass of a banana is 50 grams more than the mass of the apple.

The total mass of the plum, the apple and the banana is 450 grams.

Form an equation in n and solve it to find the mass of the plum.

.... g [3]

(c) Solve by factorisation.

$$x^2 - 4x - 21 = 0$$

$x =$ .... or $x =$ .... [3]

17. Nov/2023/Paper_ 4024/22/No.6

(a) (i) Complete the table for $y = \dfrac{4^x}{10}$.

x	0	0.5	1	1.5	2	2.5	3
y		0.2	0.4	0.8	1.6	3.2	6.4

[1]

(ii) Draw the graph of $y = \dfrac{4^x}{10}$ for $0 \leqslant x \leqslant 3$.
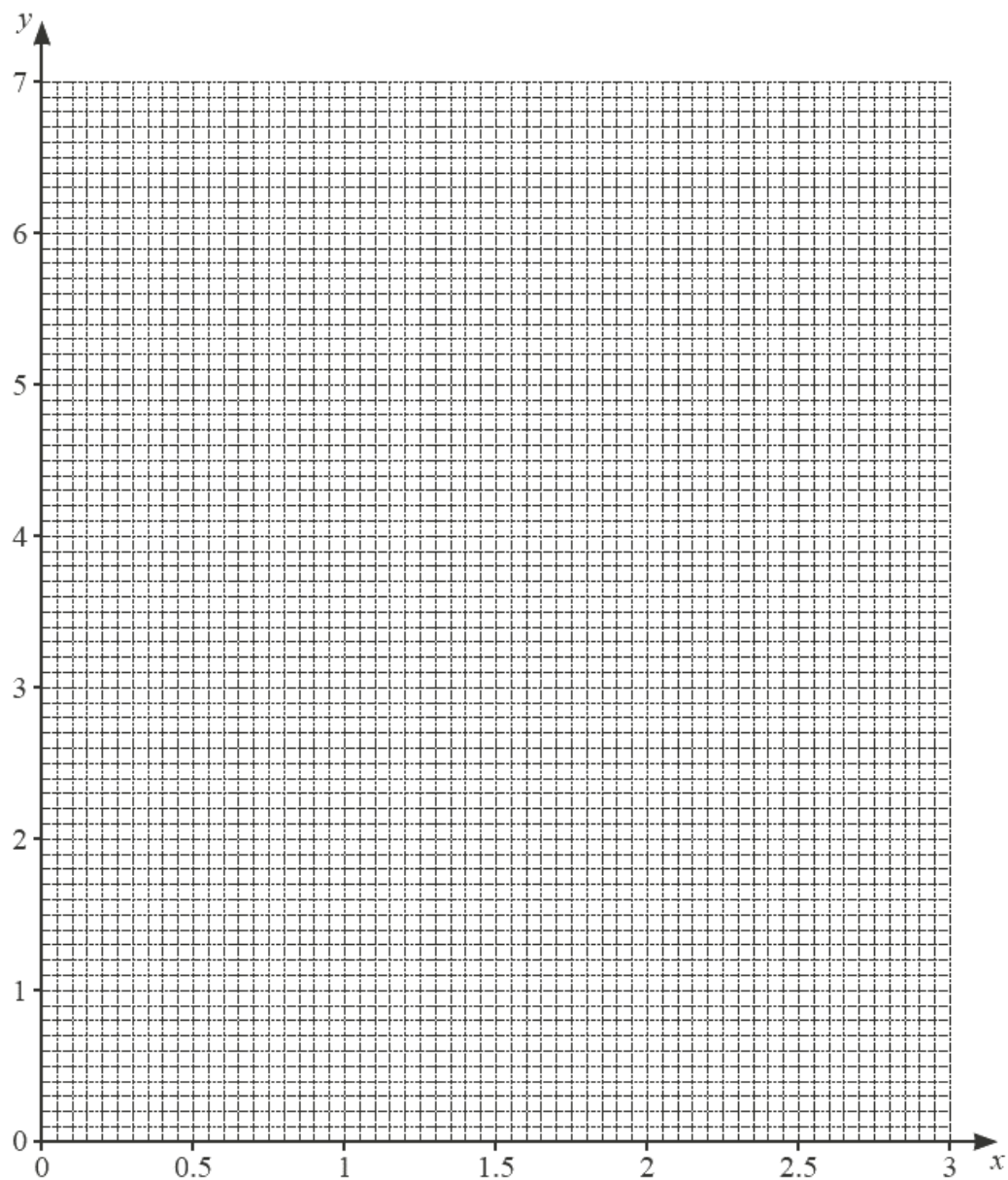
[3]

(iii) Use your graph to solve the equation $\dfrac{4^x}{10} = 5$.

$x = $ .... [1]

(iv) By drawing a tangent, estimate the gradient of $y = \dfrac{4^x}{10}$ when $x = 2$.

.... [2]

(b)
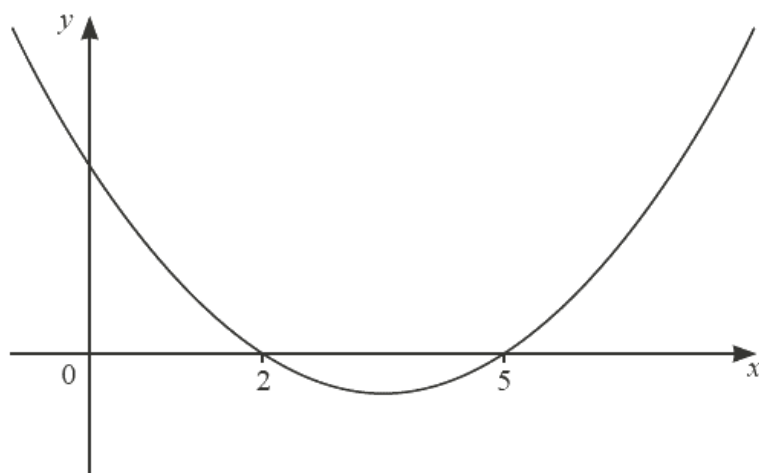
NOT TO SCALE

This is a sketch of the graph of $y = x^2 + ax + b$.

Find the value of a and the value of b.

$a = $ ....

$b = $ .... [3]

18. Nov/2023/Paper_ 4024/22/No.8
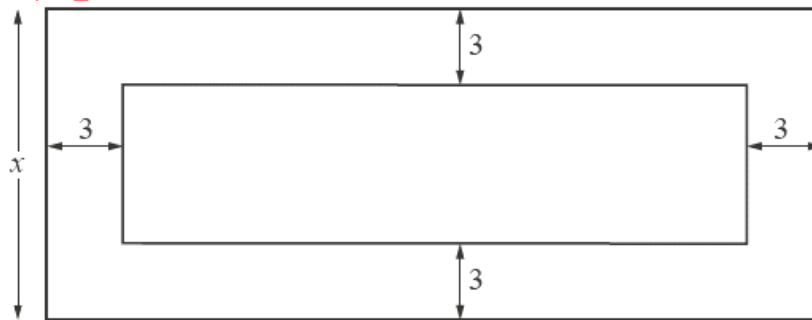
NOT TO SCALE

The diagram shows a small rectangle inside a large rectangle.

The height of the large rectangle is x cm.
The length of the large rectangle is 4 times its height.
The width of the shaded border is 3 cm.

The area of the **small** rectangle is 80 cm^2.

(a) Form an equation in x and show that it simplifies to $2x^2 - 15x - 22 = 0$.

[4]

(b) Solve the equation $2x^2 - 15x - 22 = 0$.
Show your working and give your answers correct to 2 decimal places.

$x = $ .... or $x = $ .... [3]

(c) Calculate the shaded area.

.... cm^2 [2]

19. Nov/2023/Paper_ 4024/22/No.11

$$f(x) = 4x + 1 \qquad g(x) = 2x - 3$$

(a) Find $f(-3)$.

.... [1]

(b) Find $g^{-1}(x)$.

$g^{-1}(x) = $ .... [2]

(c) Simplify $\dfrac{2}{f(x)} + \dfrac{1}{g(x)}$.

Give your answer as a single fraction, in terms of x, in its simplest form.

.... [3]

20. June/2023/Paper_ 4024/11/No.4

    (a) Benjamin's age is t years.

        (i) Maryam is 5 years younger than Benjamin.

        Write an expression for Maryam's age in terms of t.

        .... [1]

        (ii) Colin's age is twice Benjamin's age.

        Write an expression for Colin's age in terms of t.

        .... [1]

    (b) Given that $a = 3$ and $b = -2$, evaluate $5a - 2b$.

        .... [1]

21. June/2023/Paper_ 4024/11/No.11

    Solve the inequality $x - 5 > 3x + 7$.

        .... [2]

22. June/2023/Paper_ 4024/11/No.14

Expand and simplify.

(a) $5(3x-2)-3(2x-3)$

.... [2]

(b) $(2x+3)(x-7)$

.... [2]

23. June/2023/Paper_ 4024/11/No.15

These are the first four terms of a sequence.

$$1 \qquad 7 \qquad 13 \qquad 19$$

Find an expression, in terms of n, for the nth term of this sequence.

.... [2]

24. June/2023/Paper_ 4024/11/No.18

$$y = \sqrt{\frac{x+2}{3}}$$

Rearrange the formula to make x the subject.

$x =$ .... [3]

25. June/2023/Paper_ 4024/11/No.20

$$f(x) = 10 + 7x$$

Find $f^{-1}(x)$.

$f^{-1}(x) =$ .... [2]

26. June/2023/Paper_ 4024/11/No.22

Factorise.

(a) $7y + 2xy - 6x - 21$

.... [2]

(b) $3a^2 - 12b^2$

.... [2]

27. June/2023/Paper_ 4024/11/No.25

$(ax^b)^3 = 27x^4$

Find the value of a and the value of b.

$a = $ ....

$b = $ .... [2]

28. June/2023/Paper_ 4024/12/No.10

Solve the simultaneous equations.
Show your working.

$$x + 2y = 7$$
$$3x + 4y = 11$$

$x =$ ....

$y =$ .... [3]

29. June/2023/Paper_ 4024/12/No.13

(a) These are the first four terms of a sequence.

1   3   9   27

Find the next term of the sequence.

.... [1]

(b) These are the first five terms of a different sequence.

35   31   27   23   19

Find an expression, in terms of n, for the nth term of this sequence.

.... [2]

30. June/2023/Paper_ 4024/12/No.15

Three lines and a shaded region are shown on a 1 cm square grid.
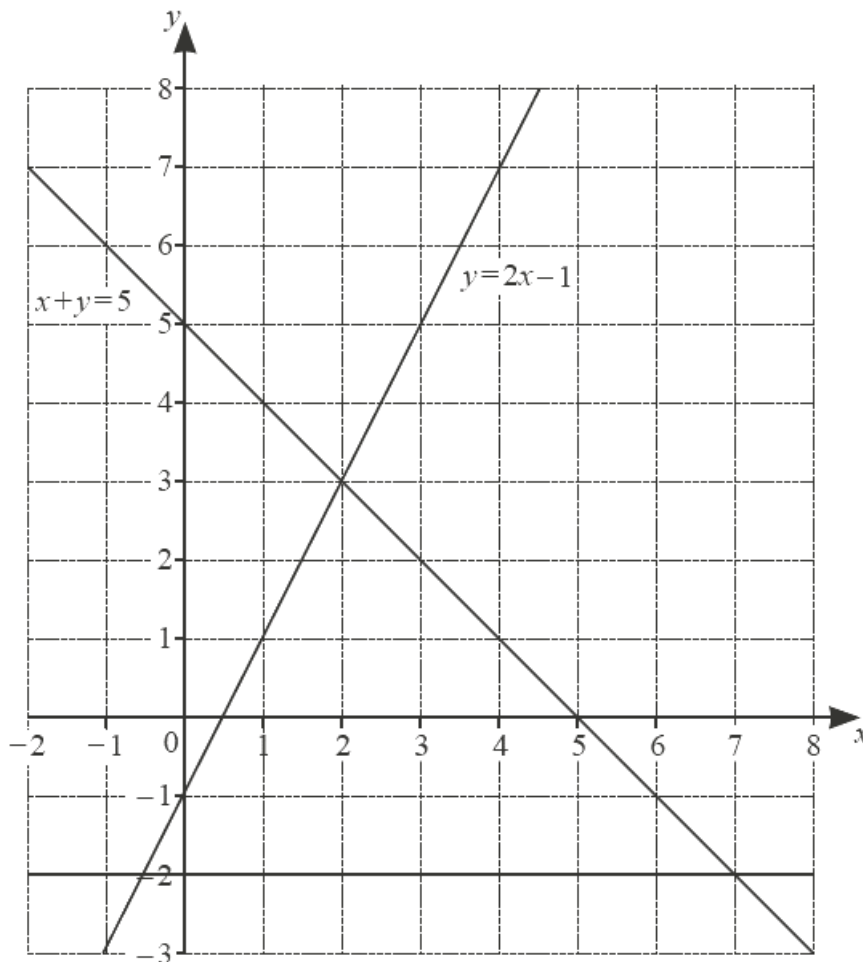
(a) Find the three inequalities that define the shaded region.

....

....

.... [2]

(b) Another region, R, is defined by these three inequalities.

$$x+y \leqslant 5 \qquad y \geqslant 2x-1 \qquad x \geqslant 1$$

Find the area of region R.

.... cm^2 [1]

31. June/2023/Paper_ 4024/12/No.17

$f(x) = 2 - 3x$ $g(x) = x - 4$

(a) Find $f^{-1}(x)$.

$f^{-1}(x) =$ .... [2]

(b) Solve $f(x + 5) = 3g(x)$.

$x =$ .... [3]

32. June/2023/Paper_ 4024/12/No.20

(a) Expand and simplify.

$$(4x - y)(2x + 5y)$$

.... [2]

(b) Simplify.

$$\left(\frac{x^{12}}{8}\right)^{\frac{2}{3}}$$

.... [2]

33. June/2023/Paper_ 4024/12/No.21

Solve.

$$\frac{5x}{x-3} = x+4$$

$x =$ .... or $x =$ .... [4]

34. June/2023/Paper_ 4024/21/No.1(a _ c)
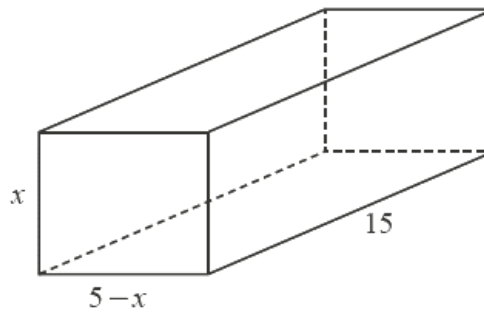
A cuboid has dimensions x cm, $(5-x)$ cm and 15 cm.

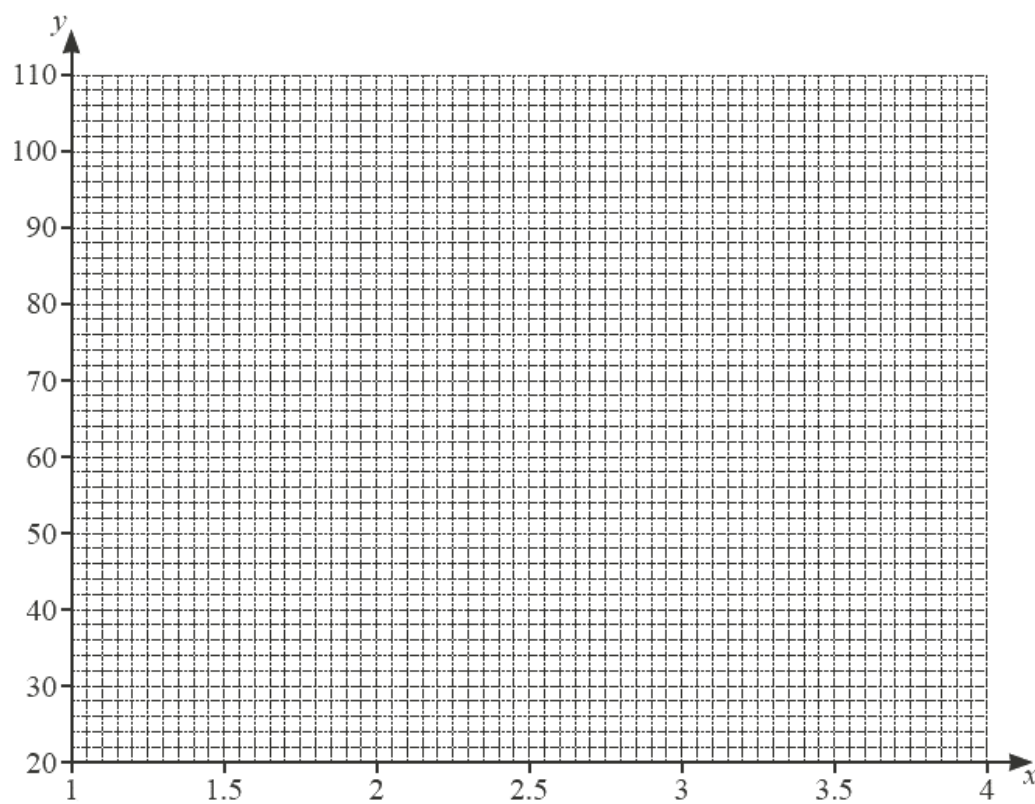**(a)** Show that the equation for the volume of the cuboid, y cm^3, is $y = 75x - 15x^2$.

[1]

(b) Complete the table of values for $y = 75x - 15x^2$.

x	1	1.5	2	2.5	3	3.5	4
y	60	78.75	90		90	78.75	60

[1]

(c) Draw the graph of $y = 75x - 15x^2$ for $1 \leqslant x \leqslant 4$.

[3]

35. June/2023/Paper_ 4024/21/No.6

(a) Simplify $3u - 6w - 5u + 9w$.

.... [2]

(b) Emilio buys m pencils at 40 cents each and 12 pens at 85 cents each.
He pays $20 and receives $2.20 change.

Form an equation in m and solve it to find the number of pencils Emilio buys.
Show your working.

.... pencils [4]

(c) y is directly proportional to the cube of $(x-2)$.
When $y = 12$, $x = 4$.

Find y when $x = 5$.

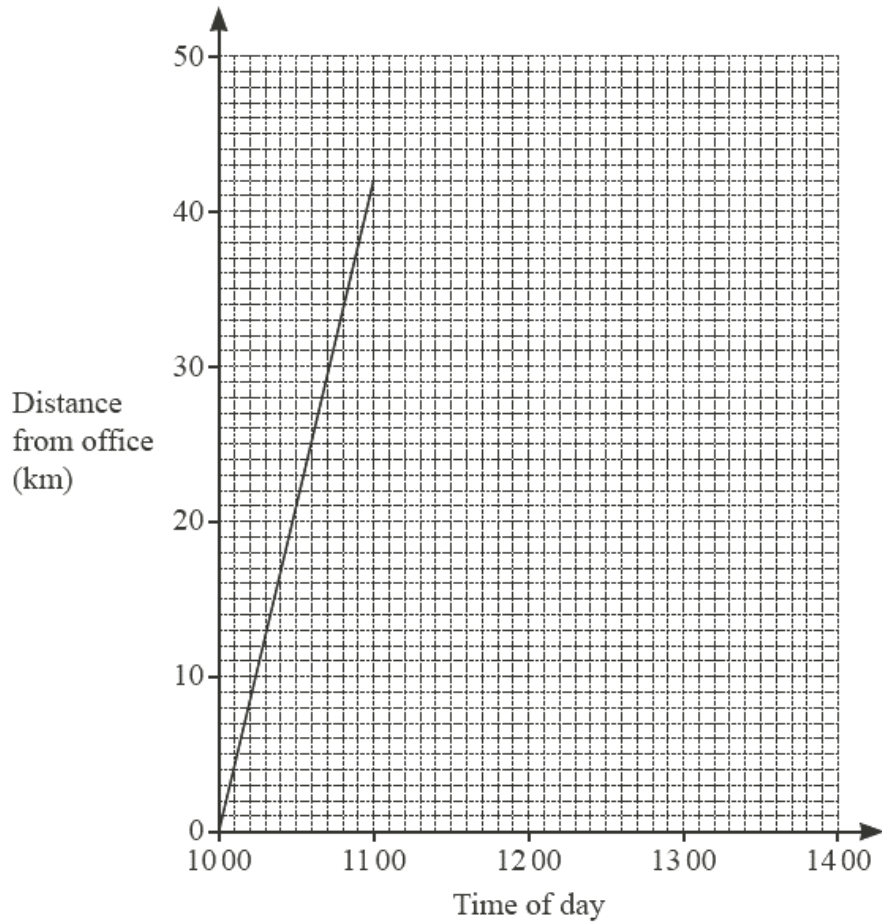$y =$ .... [2]

(d) Write as a single fraction in its simplest form.

$$\frac{3}{x-1} - \frac{4}{2x+1}$$

.... [3]

36. June/2023/Paper_ 4024/21/No.8

(a) Maya leaves the office for a factory visit.
The distance–time graph shows her journey from the office to the factory.

(i) Maya stays at the factory for $1\frac{1}{2}$ hours.

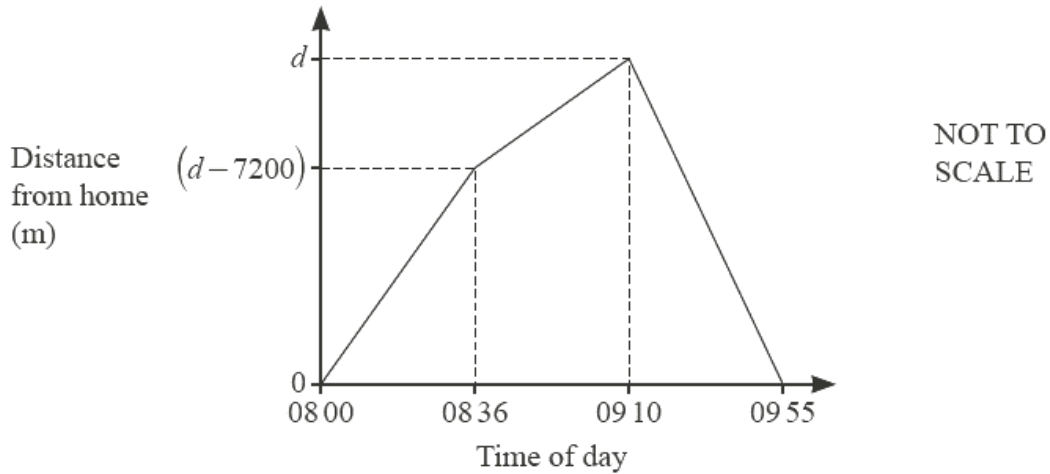She then returns to the office at an average speed of 35 km/h.

Complete the distance–time graph to show this information. [2]

(ii) Use your graph to find the time Maya arrives back at the office.

.... [1]

(b) The diagram shows the distance–time graph for the journey of a cyclist.
The cyclist travels *d* metres from home to a lake and then returns home.

At 08 36 the cyclist is $(d-7200)$ metres from home.

The average speed of the cyclist between 08 00 and 08 36 is $\frac{4}{5}$ of the average speed of the cyclist between 09 10 and 09 55.

Calculate the value of *d*.

$d =$ .... [3]

37. June/2023/Paper_ 4024/22/No.2

(a) One chocolate bar costs p cents and one packet of sweets costs 75 cents.
Tanish pays \$9.10 for 5 chocolate bars and 8 packets of sweets.

Form an equation and solve it to find the value of p.
Show your working.

$p =$ .... [3]

(b) Factorise $6ac - 27c$.

.... [2]

(c) Write $\dfrac{3m^2n}{10} \times \dfrac{5n}{9m}$ as a single fraction in its simplest form.

.... [2]

(d) Rearrange the formula $y = \dfrac{3x^2}{5}$ to make x the subject.

$x =$ .... [2]

(e) A group of k numbers has a mean of 56.8 .
The number 52 is added to the group.
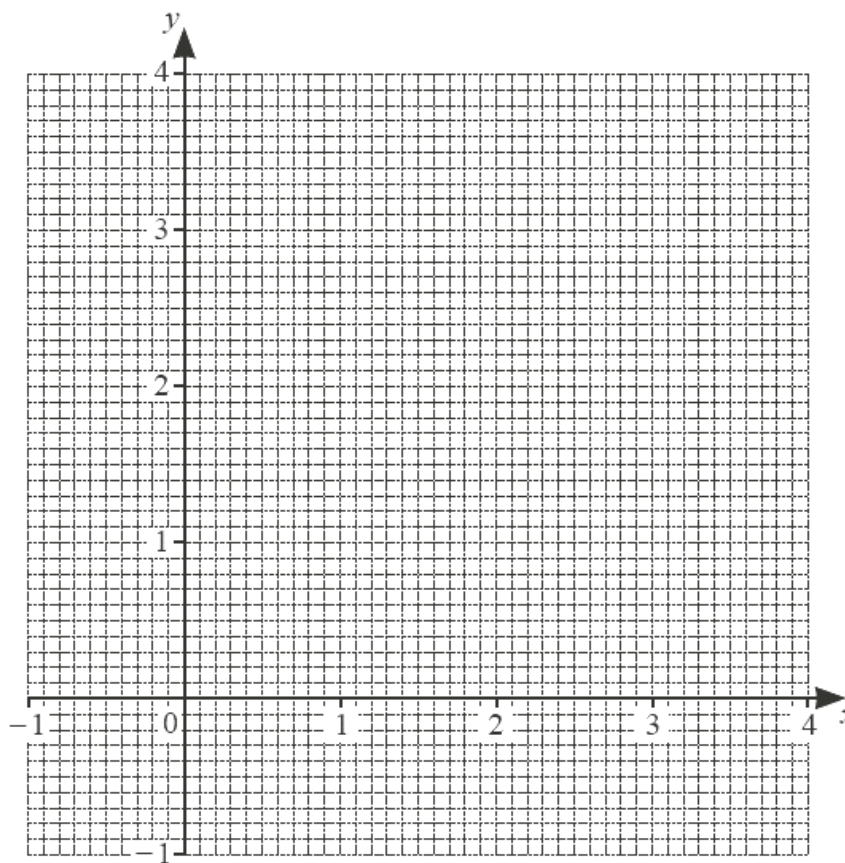The new mean is 56.5 .

Find the value of k.

$k =$ .... [4]

38. June/2023/Paper_ 4024/22/No.4

(a) Complete the table of values for $y = \dfrac{2^x}{5}$.

x	-1	0	1	2	3	4
y	0.1	0.2	0.4	0.8	1.6	

[1]

(b) Draw the graph of $y = \dfrac{2^x}{5}$ for $-1 \leqslant x \leqslant 4$.

[3]

(c) By drawing a suitable line on the grid, solve $2^x = 6$.

$x = $ .... [2]

(d) (i) Complete the table of values for $4y = 2x + 1$.

x	-1	2	4
y			2.25

[1]

(ii) On the grid on page 8, draw the graph of $4y = 2x + 1$ for $-1 \leqslant x \leqslant 4$. [1]

(iii) Find the x-coordinates of the points where the line $4y = 2x + 1$ crosses the graph of $y = \dfrac{2^x}{5}$.

$x = $ .... and $x = $ .... [1]

(iv) The x-coordinates in **part (d)(iii)** are the solutions of the equation $A \times 2^x + Bx + C = 0$, where A, B and C are all integers.

Use the equations $4y = 2x + 1$ and $y = \dfrac{2^x}{5}$ to find the exact value of each of A, B and C.

$A = $ ....

$B = $ ....

$C = $ .... [3]