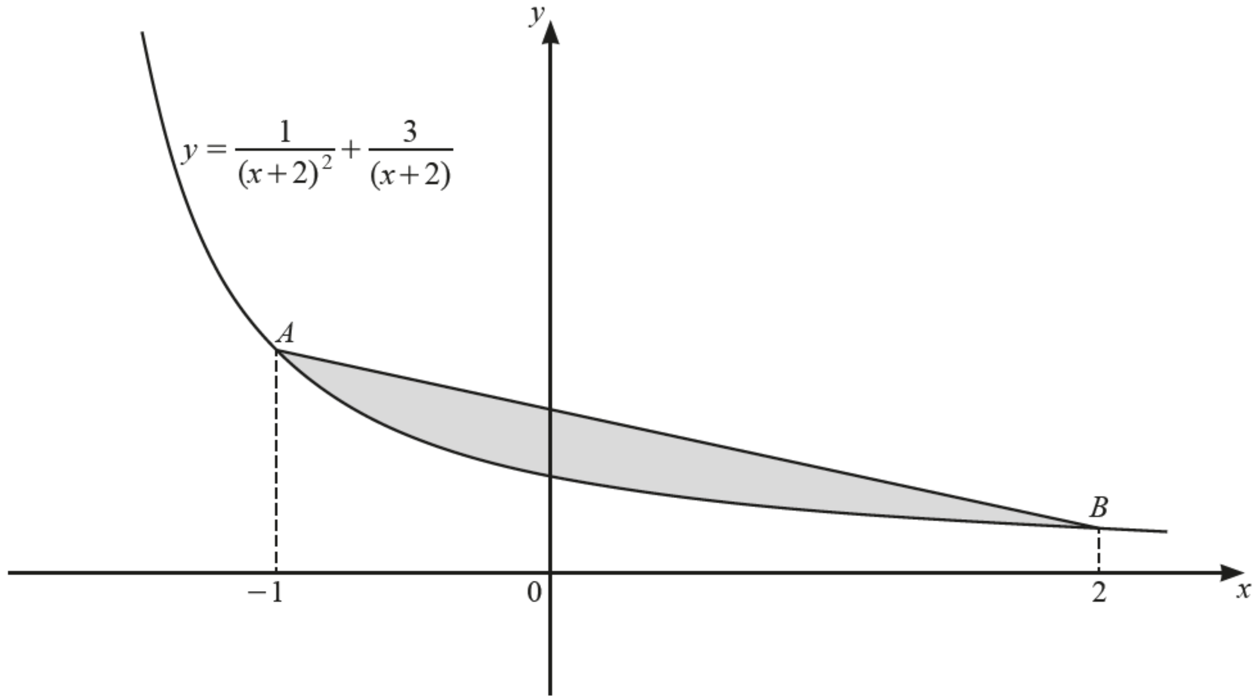
Differentiation and integration – 2021 O Level Additional Math

1. Nov/2021/Paper_12/No.10



The diagram shows the graph of the curve $y = \frac{1}{(x+2)^2} + \frac{3}{(x+2)}$ for $x > -2$. The points A and B lie on the curve such that the x-coordinates of A and of B are -1 and 2 respectively.

(a) Find the exact y-coordinates of A and of B. [2]

(b) Find the area of the shaded region enclosed by the line AB and the curve, giving your answer in the form $\frac{p}{q} - \ln r$, where p, q and r are integers. [6]

2. Nov/2021/Paper_13/No.10

A curve with equation $y = f(x)$ is such that $\frac{d^2y}{dx^2} = (2x+3)^{-\frac{1}{2}} + 5$ for $x > 0$. The curve has gradient 10 at the point $\left(3, \frac{19}{2}\right)$.

(a) Show that, when $x = 11$, $\frac{dy}{dx} = 52$. [5]

(b) Find $f(x)$. [4]

3. Nov/2021/Paper_13/No.11

A curve has equation $y = \frac{(x^2 - 5)^{\frac{1}{3}}}{x+1}$ for $x > -1$.

(a) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{Ax^2 + Bx + C}{3(x+1)^2(x^2-5)^{\frac{2}{3}}}$ where A, B and C are integers. [6]

(b) Find the x-coordinate of the stationary point on the curve. [2]

(c) Explain how you could determine the nature of this stationary point.
[You are not required to find the nature of this stationary point.] [2]

4. Nov/2021/Paper_22/No.4

(a) Find the x-coordinates of the stationary points on the curve $y = 3\ln x + x^2 - 7x$, where $x > 0$. [5]

(b) Determine the nature of each of these stationary points. [3]

5. Nov/2021/Paper_22/No.7

It is given that $\dfrac{d^2y}{dx^2} = e^{2x} + \dfrac{1}{(x+1)^2}$ for $x > -1$.

(a) Find an expression for $\dfrac{dy}{dx}$ given that $\dfrac{dy}{dx} = 2$ when $x = 0$. [3]

(b) Find an expression for y given that $y = 4$ when $x = 0$. [3]

6. Nov/2021/Paper_23/No.6

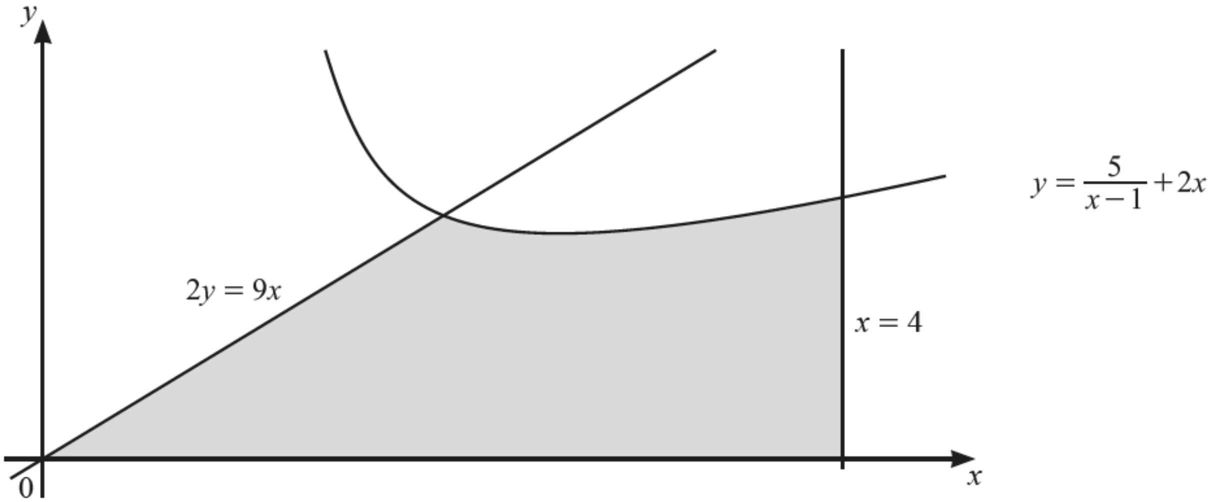
It is given that $x = 2 + \sec\theta$ and $y = 5 + \tan^2\theta$.

(a) Express y in terms of x. [2]

(b) Find $\frac{dy}{dx}$ in terms of x. [1]

(c) A curve has the equation found in **part (a)**. Find the equation of the tangent to the curve when $\theta = \frac{\pi}{3}$. [4]

7. Nov/2021/Paper_23/No.8



The diagram shows part of the curve $y = \dfrac{5}{x-1} + 2x$, and the straight lines $x = 4$ and $2y = 9x$.

(a) Find the coordinates of the stationary point on the curve $y = \dfrac{5}{x-1} + 2x$. [5]

8. June/2021/Paper_11/No.2

Find $\int_3^5 \left(\frac{1}{x-1} - \frac{1}{(x-1)^2}\right) dx$, giving your answer in the form $a + \ln b$, where a and b are rational numbers. [5]

9. June/2021/Paper_11/No.8b

(b) Find the x-coordinate of the stationary point on the curve, giving your answer in the form $a + b\sqrt{3}$, where a and b are rational numbers. [3]

10. June/2021/Paper_12/No.4

A curve is such that $\frac{d^2y}{dx^2} = (3x+2)^{-\frac{1}{3}}$. The curve has gradient 4 at the point $(2, 6.2)$. Find the equation of the curve. [6]

11. June/2021/Paper_12/No.6

DO NOT USE A CALCULATOR IN THIS QUESTION.

A curve has equation $y=(3+\sqrt{5})x^2-8\sqrt{5}x+60$.

(a) Find the x-coordinate of the stationary point on the curve, giving your answer in the form $a+b\sqrt{5}$, where a and b are integers. [4]

(b) Hence find the y-coordinate of this stationary point, giving your answer in the form $c\sqrt{5}$, where c is an integer. [3]

12. June/2021/Paper_12/No.11

The normal to the curve $y = \dfrac{\ln\left(x^2+2\right)}{2x-3}$ at the point where $x = 2$ meets the y-axis at the point P.

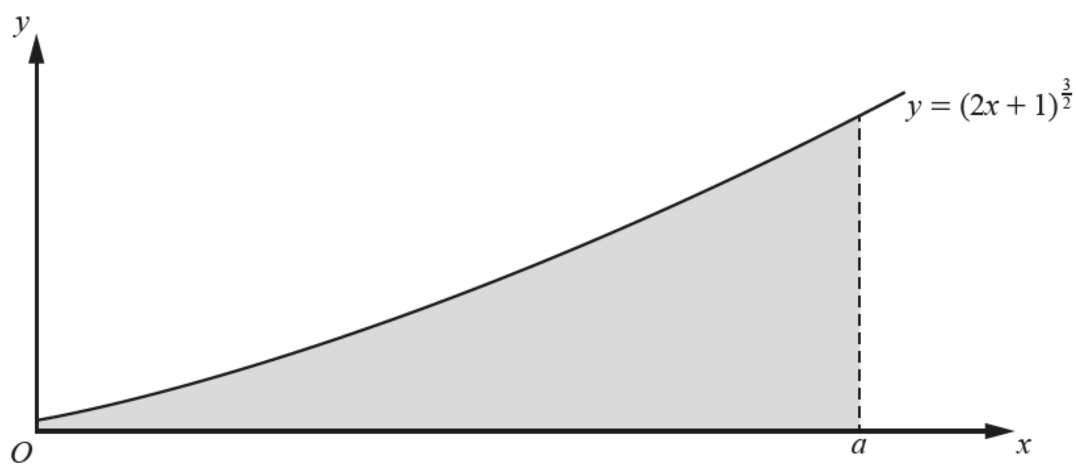
Find the coordinates of P. [7]

13. June/2021/Paper_14/No.4

(a) Find $\frac{\mathrm{d}}{\mathrm{d}x}(2x+1)^{\frac{5}{2}}$. [2]

(b) Hence find $\int (2x+1)^{\frac{3}{2}}\,\mathrm{d}x$. [2]

(c)



The diagram shows the graph of the curve $y=(2x+1)^{\frac{3}{2}}$ for $x \geqslant 0$. The shaded region enclosed by the curve, the axes and the line $x = a$ is equal to 48.4 square units. Find the value of a, showing all your working. [3]

14. June/2021/Paper_14/No.7

A curve is such that $\dfrac{d^2y}{dx^2} = 8\sin 2x$. The curve has a gradient of 6 at the point $\left(\dfrac{\pi}{2}, 4\pi\right)$.

Find the equation of the curve. [8]

15. June/2021/Paper_14/No.11

The tangent at the point where $x = 1$ on the curve $y = 6x\ln(x^2+1)$ intersects the y-axis at the point P. This tangent also intersects the line $x = 2$ at the point Q. A line through P, parallel to the x-axis, meets the line $x = 2$ at the point R. Find the exact area of triangle PQR. [10]

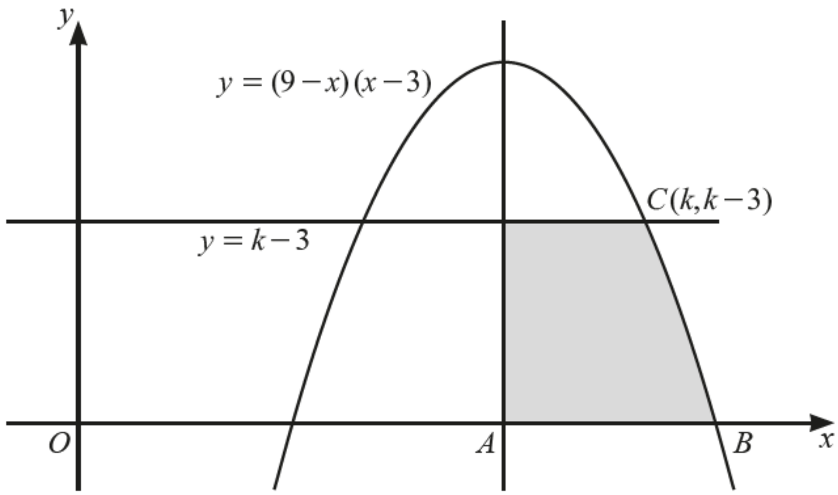
16. June/2021/Paper_21/No.6

Variables x and y are such that $y = e^{\frac{x}{2}} + x\cos 2x$, where x is in radians. Use differentiation to find the approximate change in y as x increases from 1 to $1+h$, where h is small. [6]

17. June/2021/Paper_21/No.9

A curve is such that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \sin\left(6x - \frac{\pi}{2}\right)$. Given that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}$ at the point $\left(\frac{\pi}{4}, \frac{13\pi}{12}\right)$ on the curve, find the equation of the curve. [7]

18. June/2021/Paper_21/No.12



The diagram shows part of the curve $y = (9 - x)(x - 3)$ and the line $y = k - 3$, where $k > 3$. The line through the maximum point of the curve, parallel to the y-axis, meets the x-axis at A. The curve meets the x-axis at B, and the line $y = k - 3$ meets the curve at the point $C(k,\ k - 3)$. Find the area of the shaded region. [9]

19. June/2021/Paper_22/No.8

A particle moves in a straight line so that, t seconds after passing through a fixed point O, its velocity, $v\ \mathrm{ms}^{-1}$, is given by $v = 3t^2 - 30t + 72$.

(a) Find the distance between the particle's two positions of instantaneous rest. [6]

(b) Find the acceleration of the particle when $t = 2$. [2]

20. June/2021/Paper_22/No.10

(a) Find $\int (e^{x+1})^3 dx$. [2]

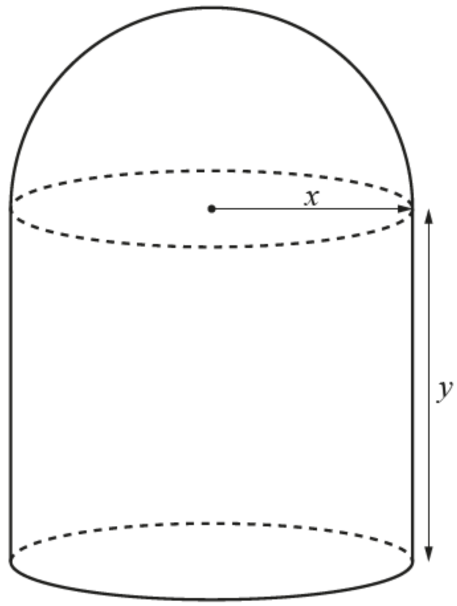
(b) (i) Differentiate, with respect to x, $y = x\sin 4x$. [2]

(ii) Hence show that $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} 4x\cos 4x dx = \frac{1}{8} - \frac{\pi\sqrt{3}}{6}$. [4]

21. June/2021/Paper_22/No.11

In this question all lengths are in centimetres.

The volume and surface area of a sphere of radius r are $\frac{4}{3}\pi r^3$ and $4\pi r^2$ respectively.



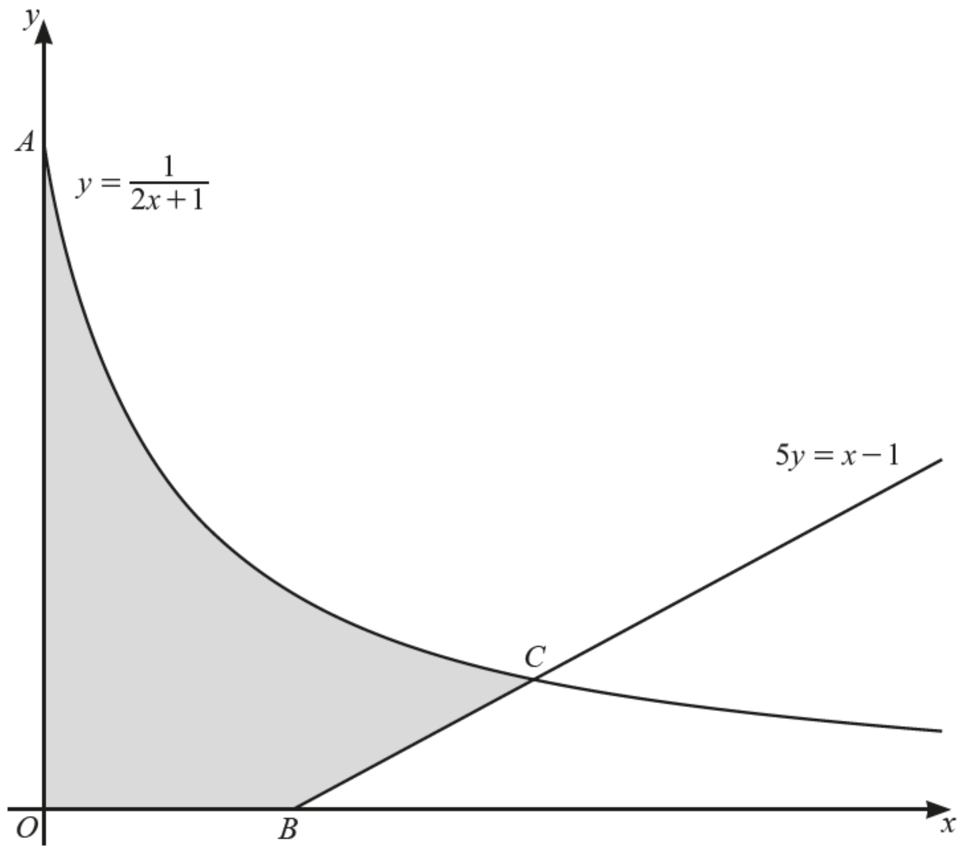
The diagram shows a solid object made from a hemisphere of radius x and a cylinder of radius x and height y. The volume of the object is $500\,\text{cm}^3$.

(a) Find an expression for y in terms of x and show that the surface area, S, of the object is given by $S = \frac{5}{3}\pi x^2 + \frac{1000}{x}$. [4]

(b) Given that x can vary and that S has a minimum value, find the value of x for which S is a minimum. [4]

22. June/2021/Paper_22/No.12

DO NOT USE A CALCULATOR IN THIS QUESTION.



The diagram shows part of the curve $y = \frac{1}{2x+1}$ and part of the line $5y = x - 1$.

The curve meets the y-axis at point A. The line meets the x-axis at point B. The line and curve intersect at point C.

(a) **(i)** Find the coordinates of A and B. [1]

(ii) Verify that the x-coordinate of C is 2. [2]

(b) Find the exact area of the shaded region. [5]

23. June/2021/Paper_24/No.2

Find $\int\left(\frac{1}{2x-3}+\sqrt{x}\right)\mathrm{d}x$. [3]

24. June/2021/Paper_24/No.4

The normal to the curve $y = x^5 - 2x^3 + x^2 + 3$ at the point on the curve where $x = -1$, cuts the x-axis at the point P. Find the equation of the normal and the coordinates of P. [7]

25. June/2021/Paper_24/No.6

The variables x and y are such that $y = \sqrt[3]{x^3 - 91}$.

(a) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$. [2]

(b) Hence, find the approximate change in y as x increases from 6 to $6+h$, where h is small. [2]

26. June/2021/Paper_24/No.10

(a) A particle P travels in a straight line so that, t seconds after passing through a fixed point O, its displacement, s metres from O, is given by

$$s = \frac{31}{3} - \frac{e^t}{3} - 10e^{-t}.$$

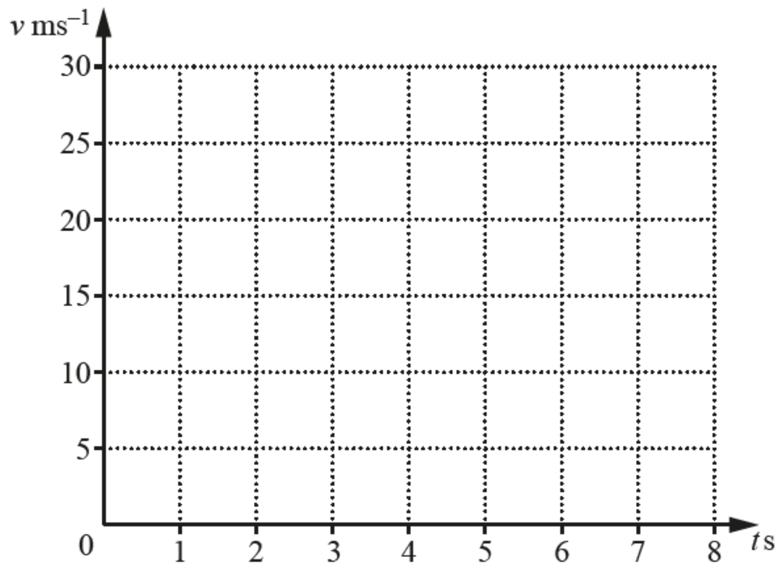
(i) Find the value of t when P is at instantaneous rest, giving your answer correct to 2 significant figures. [4]

(ii) Find the distance travelled in the first two seconds. [3]

(b) A particle Q travels in a straight line so that t seconds after leaving a fixed point O, its velocity, $v\ \mathrm{ms}^{-1}$, is given by

$$v = 2t \quad \text{for } 0 \leqslant t \leqslant 5\ ,$$
$$v = t^2 - 8t + 25 \quad \text{for } t > 5\ .$$

(i) On the axes below, sketch the velocity-time graph for the first 8 seconds of the motion of particle Q. [2]



(ii) Showing all your working, find the distance travelled by Q in the first 8 seconds of its motion. [5]